

Compton effect (Item No.: P2524415)

Curricular Relevance



Difficulty



Difficult

Preparation Time



1 Hour

Execution Time



2 Hours

Recommended Group Size



2 Students

Additional Requirements:

- PC

Experiment Variations:

Keywords:

Interactions of photons with electrons, conservation of momentum and energy, inelastic and elastic scattering, interaction cross section, Compton wavelength

Overview

Short description

Principle

When photons are scattered on electrons, their momentum and energy gets changed. The change of energy in dependence on scattering angle is measured by γ -spectroscopy. The Compton wavelength is determined from the measurement data.



Equipment

Position No.	Material	Order No.	Quantity
1	Multichannel analyser	13727-99	1
2	measure Software multi channel analyser	14452-61	1
3	Radioactive source Cs-137, 37 kBq	09096-01	1
4	Radioactive source Cs-137, 18.5 MBq	09096-20	1
5	Radioactive source Na-22, 74 kBq	09047-52	1
6	Gamma detector	09101-00	1
7	PHYWE High Precision Power Supply 1.5 kV DC	09107-99	1
8	Screening cylinder for gamma detector	09101-11	1
9	Rod, iron, d 25 mm, l 200 mm	09101-13	1
10	Lead block, 200x100x50 mm	09029-11	1
11	Lead brick with hole	09021-00	1
12	Source holder on fixing magnet	09202-00	1
13	Screened cable, BNC, l = 750 mm	07542-11	1
14	High-voltage connecting cable	09101-10	1

Tasks

1. Calibrate the set-up with the aid of the ^{137}Cs (34 kBq) and the ^{22}Na (74 kBq) sources.
2. Measure spectra of the radiation scattered by a steel rod at various angles.
3. Calculate the Compton wavelength from the spectra.

Set-up and Procedure

Set up the experiment as shown in Fig. 1. Before turning on the operating unit for the scintillation counter, connect the high voltage cable correctly to operating unit and photomultiplier and read the instructions in the manual of the gamma-detector. Set the voltage of the operating unit to 1.0 kV . Connect the MCA to the computer's USB port and start the "measure" program. Select the Gauge "Multi Channel Analyzer" and you will receive the start window, as shown in Fig. 2.

1. Select "Spectra recording" (see Fig. 2), set the "Gain" to "Level 1", set the "Offset" to 0 %, choose "Channel number" as x-Data (see Fig. 3) and start data recording. Put the ^{137}Cs pellet into the slot of the screening cylinder for the gamma-detector near to the detector. Increase the high voltage on the detector operating unit, until the 662 keV peak gets to about channel 3500. If you are not sure about the high voltage setting, start with low settings and increase the high voltage, until the 662 keV peak wanders to the right end of the spectrum. Cancel the measurement and leave the high voltage setting unchanged from now on.

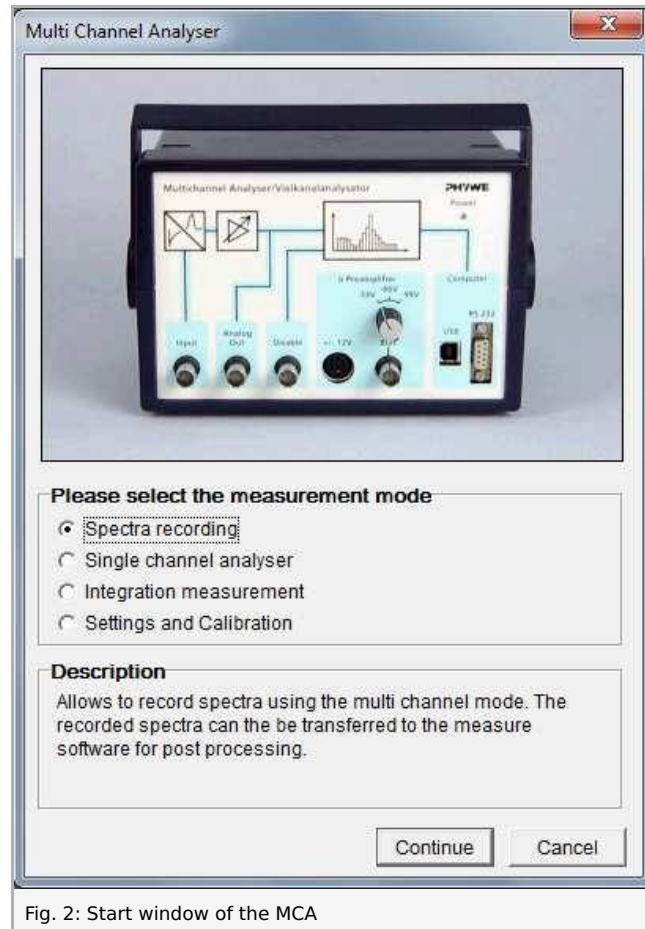


Fig. 2: Start window of the MCA

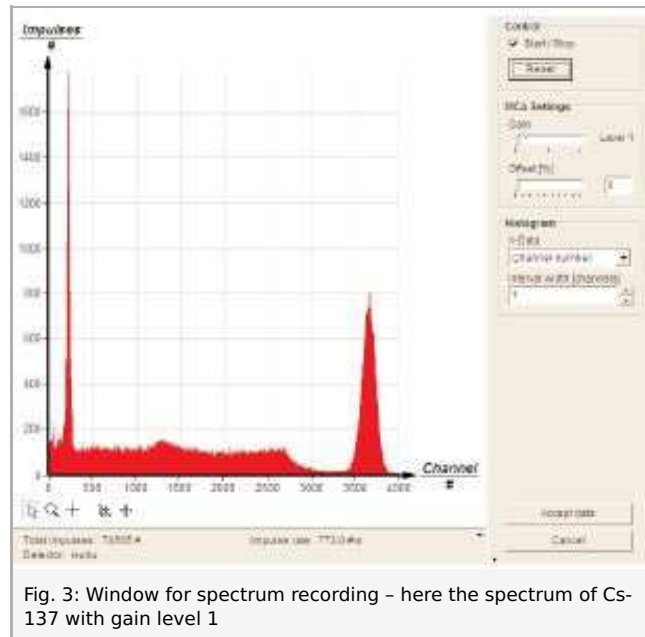


Fig. 3: Window for spectrum recording - here the spectrum of Cs-137 with gain level 1

Now start the MCA gauge again and select "Settings and calibration". The window shown in Fig. 4 will appear. Click on the "Calibrate" button. Select 3-point calibration and start with the ^{22}Na . Move two of the three bars to the 511 keV and 1275 keV peaks and type the energy values in the appropriate fields. Then remove the ^{22}Na and bring the ^{137}Cs source in direct vicinity of the detector. Click on "clear diagram" button and move the third bar to the then appearing 32.2 keV peak of the ^{137}Cs and type the energy value in the appropriate field. Finally, click on the "Apply" button and then the "Save" button of the calibration window shown in Fig. 4, and enter name and comment for your calibration as shown in Fig. 5. Remove the 34 kBq source from the slit of the cylindrical detector screening. Be careful with detector handling - the scintillation crystal breaks easily and cracks in it will reduce sensitivity and energy resolution.

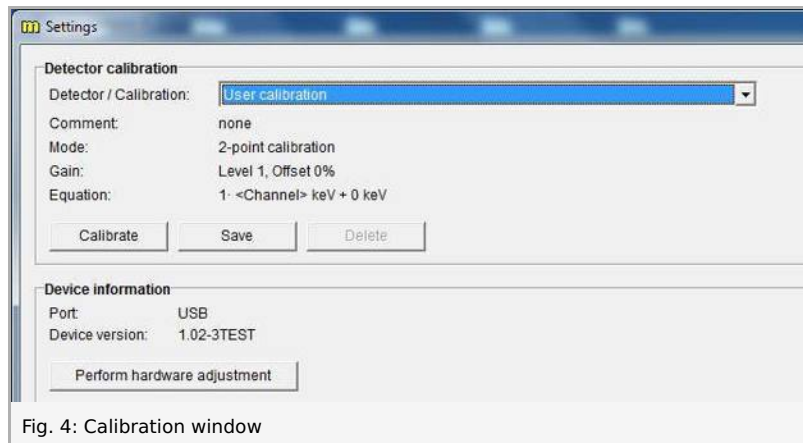


Fig. 4: Calibration window

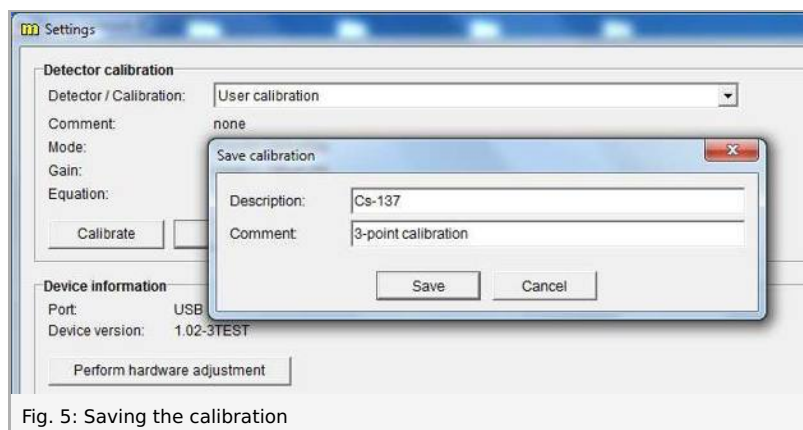


Fig. 5: Saving the calibration

- Now return to "Spectra recording" and set the "x-data" to "Energy". Put the 18.5 MBq source into the lead brick with hole. Adjust the setup in a way that mainly scattered photons from the steel rod get recorded. The 622 keV line should be well suppressed by the shielding. The presence of the steel rod should increase the impulse rate distinctly - if you take it away, it should drop clearly. The spectrum should show a distinct maximum from the preferred Compton scattering angle. Record such a spectrum for each angle (e.g. for 30°, 60°, 90°, 120°) and save the data with the "Accept data" button. Start with high or medium angles.
- Save the recorded spectra to your disc drive. Different ways may be suitable for finding the maximum of the Compton scattering peak: You may use "Analysis" > "Smooth..." to reduce the statistical noise in the spectrum and evaluate the peak position with the "Survey" tool. Else you may mark the interesting region of the spectrum with the "Mark" tool that looks like a cross and then use "Analysis" > "Function fitting..." to fit a scaled normal distribution with offset to the data (see Fig. 6) and then use "Analysis" > "Show extrema..." to find the position of the peak of the fit. Plot the Compton scattering peak energy versus the scattering angle. Plot the wavelength $\lambda = hc/E$ that corresponds to the energy E over the scattering angle and the wavelength shift, too.

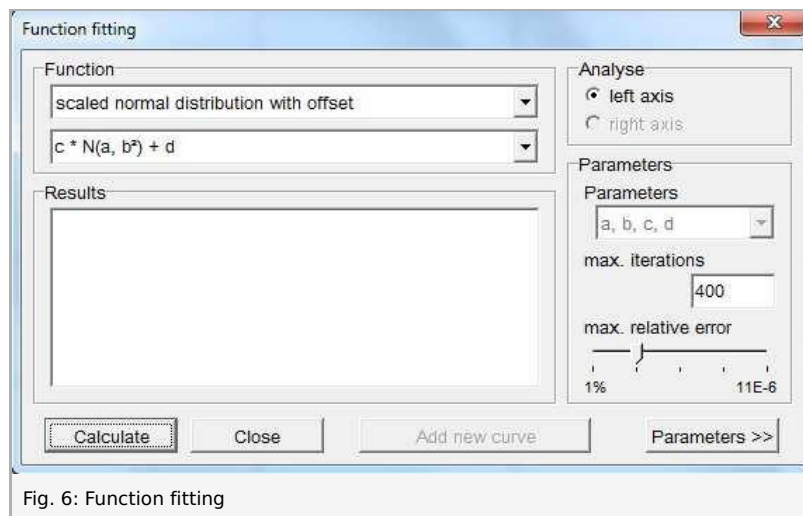


Fig. 6: Function fitting

Theory and evaluation

Compton scattering is elastic scattering of photons on electrons. Both momentum and energy conservation are fulfilled and momentum and energy are exchanged between photon and electron the same way as one would expect of mass points. The energy range is that of the electron's resting mass so the calculation has to be relativistic.

The energy E of a photon depends on its frequency f and the Planck constant h and is

$$E = hf$$

and the momentum p of a photon with wavenumber k is

$$p = \hbar k = \frac{hf}{c}$$

If the electron is at rest at the beginning, all the momentum comes from the photon and stays constant throughout the reaction. After the reaction the initial momentum is split up into the momentum of the photon. The photon moves now under the angle φ to its initial direction and has the altered frequency f' . The momentum of the electron after the scattering is $p_e = mv$ with the mass as a function of velocity ($m \geq m_e$). The vector sum of both equals the initial momentum and thus holds for the squares of the vectors:

$$m^2 v^2 = \frac{\hbar^2}{c^2} (f'^2 + f^2 - 2f' \cdot f \cdot \cos \varphi) \quad (1)$$

The energy has to be constant throughout the reaction, too and for simplicity counting the electron resting mass equivalent in the equation (the point of zero energy is arbitrary)

$$hf = m_e c^2 = hf' + mc^2 \quad (2)$$

with the relativistic relation for electron speed v

$$m_e^2 = m^2 \left(1 - \left(\frac{v}{c} \right)^2 \right) \quad (3)$$

$$(3) \Rightarrow m^2 v^2 c^2 = m^2 c^4 - m_e^2 c^4$$

$$\text{in (1)} \Rightarrow m^2 c^4 - m_e^2 c^4 = \hbar^2 (f'^2 + f^2 - 2f' \cdot f \cdot \cos \varphi) \quad (4)$$

$$(2) \Rightarrow m^2 c^4 - m_e^2 c^4 = \hbar^2 (f - f')^2 + 2m_e m c^4 - 2m_e c^4 \quad (5)$$

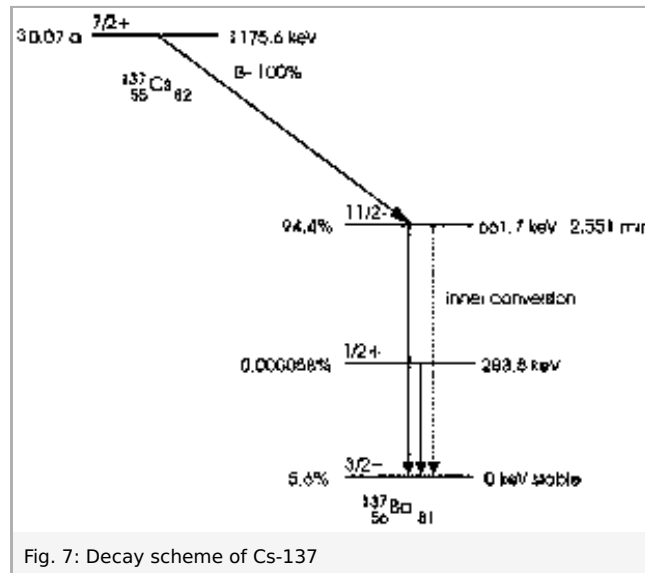
$$(4) = (5) \Rightarrow \hbar^2 f f' (1 - \cos \varphi) = m_e (m - m_e) c^4 = m_e c^2 \cdot E_{\text{kin,el}}$$

$$(2) \Rightarrow \hbar^2 f f' (1 - \cos \varphi) = m_e c^2 \hbar (f - f')$$

with $f = \frac{c}{\lambda}$, $f' = \frac{c}{\lambda'}$ and $\Delta\lambda = \lambda' - \lambda = \frac{c(f-f')}{ff'}$ is

$$\Delta\lambda = \frac{\hbar}{m_e c} (1 - \cos \varphi) \quad (6)$$

which is Compton's formula for the wavelength shift.



The wavelength

$$\frac{h}{m_e c} = \lambda_e$$

is called Compton wavelength.

With

$$E = hf = \frac{hc}{\lambda}$$

is the corresponding energy $E_c = m_e c^2$ the energy equivalent of the electron's resting mass of 511 keV. So the energy scale for the Compton effect is set by the electron mass which can be interpreted in the way that an elastic scattering occurs when the photon is about as "heavy" as the electron.

Fig. 7 shows the decay scheme of the used nuclid ^{137}Cs . The proportions of the energy scale are not displayed correctly. The main emission is the sharp 662 keV line. But there is a probability of the 11/2- excited state of the daughter nuclid ^{137m}Ba to decay by inner conversion because of the high difference of spin quantum number between ground and excited state - in the case of ^{137m}Ba leading to a 32 keV X-ray line and a long half life of the excited state of two and a half minute. Only the photons reach the detector, electrons cannot leave the source pellet.

Incoming photons of a defined energy are detected in the scintillation counter by the electrons they excite in the scintillator crystal either by photo effect, where they transfer all their momentum and energy to the electron (photon absorption or completely inelastic scattering), or by Compton scattering, where momentum and energy are exchanged between photon and electron in an elastic scattering. So the measurement result of a sharp photon energy is a whole spectrum consisting of the photo peak and the Compton background. The Compton scattering the photons undergo in the detector is here not of interest - only the incidents detected by photo effect thus showing the original photon energy should be counted for as interesting incidents leaving other effects in the spectrum ignored - you are searching for the photo peak of photons Compton scattered by the steel rod. The set-up is made in a way that only those of the 662 keV photons that have undergone scattering on the steel rod in a defined angle reach the detector and can be counted and their energy is evaluated in the recorded g-spectrum. The dependence of the energy E' of the scattered photons that had reached the steel rod with energy E_0 (here 662 keV) on the scattering angle is then from (6):

$$E' = \frac{1}{\frac{1}{m_e c^2} (1 - \cos \varphi) + \frac{1}{E_0}}$$

This function has a maximum for $\varphi = 180^\circ$, where no energy is transferred to the electron and the photon keeps its initial energy and a minimum for $\varphi = 0^\circ$, where the fraction of the initial energy

$$\frac{E'}{E_0} = \frac{1}{\frac{E_0}{m_e c^2} + 1}$$

is reflected.

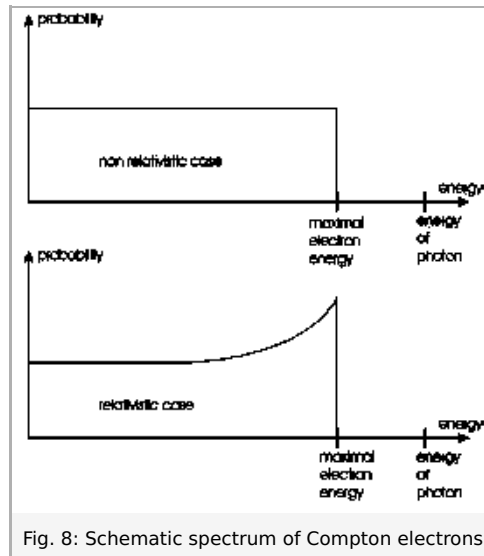


Fig. 8: Schematic spectrum of Compton electrons

For the spectrum of the electrons exited in the steel rod by Compton effect, though not measured, the following can be said: In the case of Compton scattering there is a highest energy fraction of the initial energy of the photon that can be transferred to the electron. This energy is transferred, if the photon gets reflected back into the direction where it came from (back scattering). In the non-relativistic case all scattering angles have equal probability so all the other electron energies are of equal probability if not filtered for angle, too - i.e. the spectrum of the electron energies is flat from zero up to a maximum energy, where it drops to zero. To each electron energy corresponds a defined scattering angle. In the relativistic case backscattering is preferred compared to other angles and the electron spectrum shows a rise to the maximum energy (see Fig. 4) - the precise form predicted by the Klein-Nishina formula.