Half life and radioactive equilibrium (Item No.: P2520102)

Curricular Relevance

Area of Expertise: Physics

Education Level: University

Topic: Modern Physics

Subtopic: Nuclear Physics - Radioactivity

Experiment: Half life and radioactive equilibrium

Difficulty	Preparation Time	Execution Time	Recommended Group Size
Difficult	1 Hour	2 Hours	2 Students

Additional Requirements:

Experiment Variations:

Keywords:

parent substance, daughter substance, rate of decay, disintegration or decay constant, counting rate, half life, disintegration product

Overview

Short description

Principle

The half-life of a $^{137m}\mathrm{Ba}$ daughter substance eluted (washed) out of a $^{137}\mathrm{Cs}$ isotope generator is measured directly and is also determined from the increase in activity after elution.

Experimental set-up

Equipment

Position No.	Material	Order No.	Quantity
1	Isotope generator Cs-137, 370 kBq	09047-60	1
2	Geiger-Mueller counter	13606-99	1
3	Geiger-Mueller counter tube, 15 mm (type B)	09005-00	1
4	Glass beaker DURAN®, short, 250 ml	36013-00	2
5	Test tubes 100x12 mm,FIOLAX,100pc	36307-10	1
6	Rubber stopper,d=14.5/10.5mm, w/o	39253-00	1
7	Base plate for radioactivity	09200-00	1
8	Counter tube holder on fixating magnet	09201-00	1
9	Plate holder on fixing magnet	09203-00	1
10	Source holder on fixing magnet	09202-00	1

Tasks

1. Measure the activity of the isotope generator as a function of time immediately after elution.
2. Measure the activity of a freshly eluted solution of $^{137m}\mathrm{Ba}$ as a function of time.

Set-up and procedure

Set up the experiment as shown in Fig. 1.

Procedure

In accordance with the instructions the isotope generator is eluted into a glass beaker which should then be placed as far away from the counter tube as possible. Make a U-shaped cap from a strip of aluminium sheet and put it over the tube: it will absorb the electrons in the betadecay phase which would otherwise interfere with the experiment. To measure the increase in activity it is advisable to read off the impulse rate every 30 seconds after elution (counting ratemeter time constant = 10 seconds). Activity and impulse count are sufficiently proportional at low impulse rates.

To measure the half-life of the $^{137m}\mathrm{Ba}$ isotope, first elute the isotope generator in a test tube, then place it as far away from the rest of the equipment as possible. The counter tube (without the aluminium cap) can now be set up immediately in front of the bottom end of the test tube.

Results

The isotope generator contains $400\,\mathrm{kBq}$ of $^{137}\mathrm{Cs}$, which serves as the parent substance; its half-life is 30.25 years. $^{137}\mathrm{Cs}$ decays into the barium isotope $^{137}\mathrm{Ba}$ with the emission of β-radiation. This transi-tion occurs, in part, directly (approximately $5\,\%$) to the stable ground state of $^{137}\mathrm{Ba}$ and, in part (approx-imately $95\,\%$) via the meta stable state of $^{137m}\mathrm{Ba}$. $^{137m}\mathrm{Ba}$ decays with a half-life of only $2.6\,\mathrm{min}$ under emission of γ-radiation ($E_\gamma = 662\,\mathrm{keV}$) in the stable ground state of $^{137}\mathrm{Ba}$. The decay chain is thus

$$N_1 \xrightarrow{\lambda_1} N_2 \xrightarrow{\lambda_2} N_3$$

where

$N_1 =$ number of $^{137}\mathrm{Cs}$ atoms
$N_2 =$ number of $^{137m}\mathrm{Ba}$ atoms
$N_3 =$ number of $^{137}\mathrm{Ba}$ atoms
$\lambda_1 =$ disintegration constant of $^{137}\mathrm{Cs}$
$\lambda_2 =$ disintegration constant of $^{137m}\mathrm{Ba}$.

The rates of disintegration are:

$$\frac{dN_1}{dt} = -\lambda_1 N_1$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

Solving the system of two differential equations gives for $N_2(t)$, with the initial condition that $N_2(t=0) = 0$:

$$N_2(t) = N_1(0)\frac{\lambda_1}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

The activity of the daughter substance is thus

$$A_2(t) = \lambda_2 N_2 = A_1(0)\frac{\lambda_2}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

As the half-life of the parent substance $T_{1/2}(1)$ is much greater than that of the daughter substance $T_{1/2}(2)$ in this case, then $\lambda_1 \ll \lambda_2$. If we now neglect λ_1 in relation to λ_2, we obtain

$$A_2(t) = A_1(1 - e^{-\lambda_2 t})$$

with A_1 constant.

When t is very long,

$$A_2(t) - A_1(t)$$

and

$$\lambda_2 N_2 = \lambda_1 N_1$$

i.e. the substances are in equilibrium (dynamic equilibrium, steady state).

If this equilibrium is disturbed by the removal of the daughter substance, the system will try to restore the equilibrium by forming more of it. For the increase in the daughter substance, we have:

$$A_2(t) = A_1(1 - e^{-\lambda_2 t})$$

with A_1 constant.

If the equilibrium activity

$$A_2(t = \infty) = A_1$$

Is measured prior to elution, then λ_2 can be calculated from the measured values $A - A(t)$ because

$$A_1 - A_2(t) = A_1(e^{-\lambda_2 t})$$

In the elution process, the $^{137m}\mathrm{Ba}$ is washed out of the isotope generator. Figure 2 shows the counting rate as a function of time. A counting rate that decreases exponentially over time results. The logarithmic counting rate is plotted as a function of time, the straight line given in Fig. 3 is obtained.

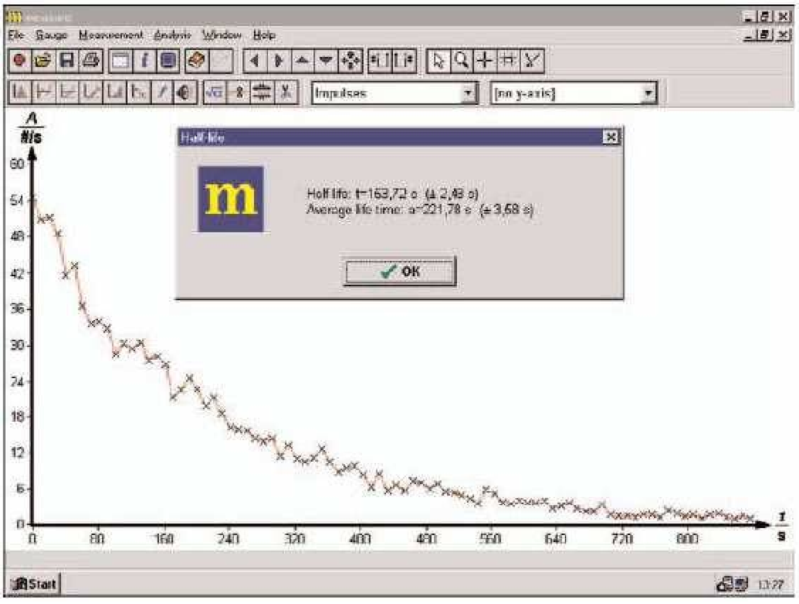

Fig. 2:Calculation of the half-life of Ba-137m’s decay, counting rate as a function of time.

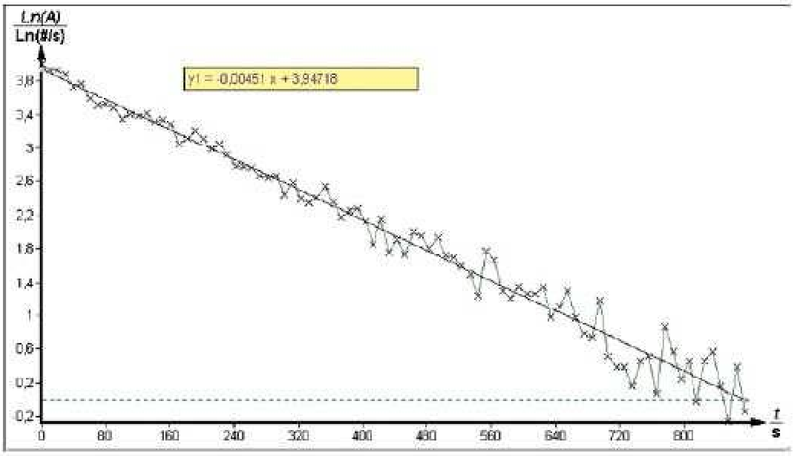

Fig. 3: Logarithmic plot of the counting rate of Ba-137m's decay; counting rate as a function of time, with the regression line.

In this case, the counting rate decreases to half of its original value after a time period $\tau = 153.69\,\mathrm{s}$. Thus, it follows that:

$$\dot{N} = \dot{N}_0\, e^{-\lambda t}$$

and

$$\frac{1}{2}\dot{N} = \dot{N}_0\, e^{-\lambda \tau}$$

where

- $\dot{N}$ Actual rate of decay
- $\dot{N}_o$ Counting rate at time
- λ Decay constant
- t Time
- τ Half-life

Thus,

$\lambda = \frac{\ln 2}{\tau}$.

By taking the logarithm of

$\dot{N} = \dot{N}_0\, e^{-\lambda t}$

the following is obtained

$\ln \dot{N} = -\lambda t \ln \dot{N}_0$

Comparing the coefficients with the regression line

$y = mtb$

shows that the slope of the regression line m corresponds to the negative decay constant

$m = -\lambda$

The decay constant λ is characteristic for the gradual decline in radioactivity: It is thus linked to the specific time period in which the substance quantity decays to half of its initial radioactivity (half-life τ).

Formation of daughter substance subsequent to disturbance of the equilibrium by elution

Calculation of the half-life. At time t an activity is measured that can be calculated according to the following formula:

$$\dot{N}(t) = \overline{\dot{N}} + \dot{N}_0\,(1 - e^{-\lambda t})$$

where

$\dot{N}(t)$ Actual counting rate

$\dot{\overline{N}}$ Activity of the β-radiating $^{137}\mathrm{Cs}$ (through the opining in the source) and the residual activity of the $^{137m}\mathrm{Ba}$, which results from elution.

$\dot{N}_0$ Equilibrium activity of the daughter substance, $^{137m}\mathrm{Ba}$

By subtracting the equilibrium activity after approximately $650\,\mathrm{s}/tex],whichiscomprisedof\backslash($ $\dot{\overline{N}}$ and $\dot{N}_0$, from the respective actual counting rate, in this case 504.28 (cf. Fig. 7) and calculating the natural logarithm of this variable's value, the diagram in Fig. 8 is obtained. The half-life for this measurement, [tex]\tau =142.33 \, \mathrm{s}\), follows from the slope of the drawn regression line

$$\tau = -\frac{\ln 2}{m} = -\frac{\ln 2}{-0.00487} = 142.33\,\mathrm{s}$$

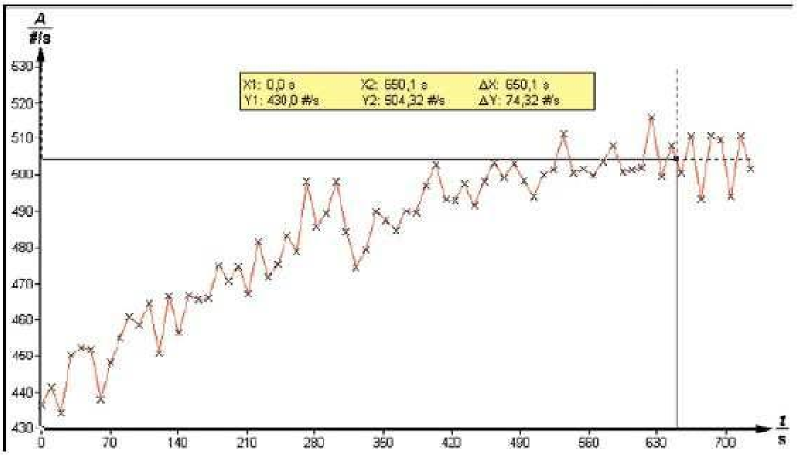

Fig 7: Plot of the counting rate of the Ba-137m being formed as a function of time.

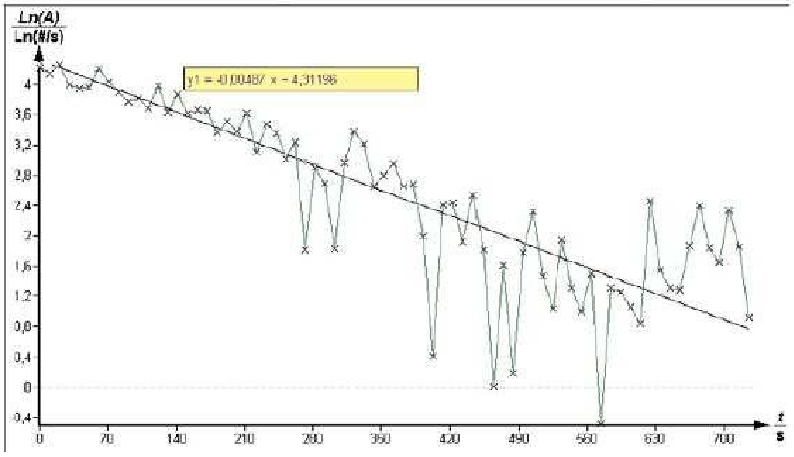

Fig 8: Logarithmic counting rateof the Ba-137m being formed as a function of time, with the regression line.

Remarks

The calculation shown above can be performed using the and func-tions using the formula ln(abs(x-504.28)). When using radioactive substances, conform absolutely to the stipulations of the respective applicable radiation protection regulations. Radioactive substances can be hazardous to your health! Always reduce the time spent handling radioactive substances to a minimum. Do not eat or drink anything in the presence of radioactive substances and always wash your hands after contact with them!