RC Filters (Item No.: P2440801)

Curricular Relevance

Area of Expertise: Physics	Education Level: University	Topic: Electricity and Magnetism	Subtopic: Inductance, Electromagnetic Oscillations, AC Circuits	Experiment: RC Filters
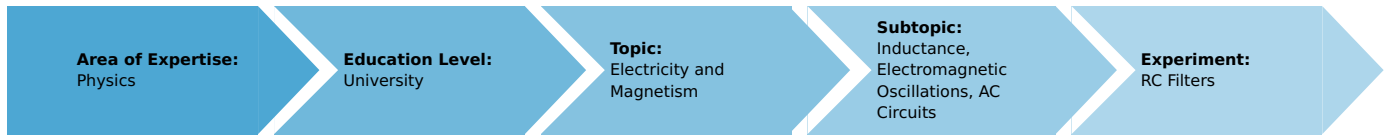
Difficulty	Preparation Time	Execution Time	Recommended Group Size
Difficult	1 Hour	2 Hours	2 Students

Additional Requirements:	Experiment Variations:

Keywords:

High-pass, low-pass, Wien-Robinson bridge, parallel-T filters, differentiating network, integrating net-work, step response, square wave, transfer function

Introduction

Overview

Resistor-Capacitor (RC) circuits serve as filters for frequencies. The frequency response of the most commonly used RC filters is recorded by point-by-point measurements as well as the frequency sweep method, and displayed on the oscilloscope. The results are plotted and verified using the "measure" analysis software.
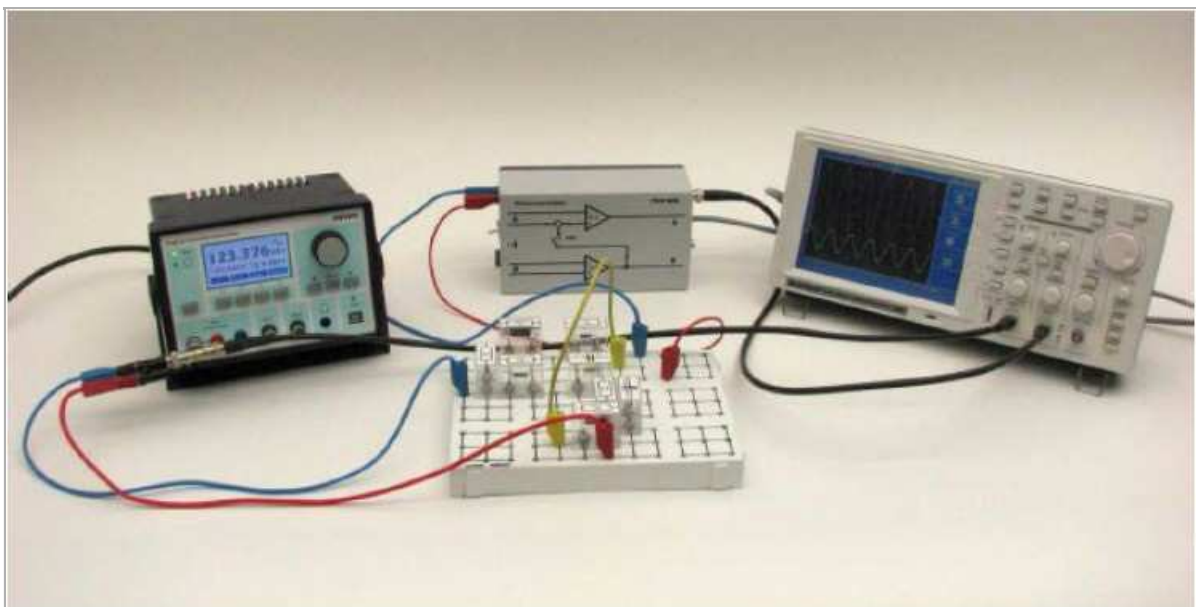

Fig. 1: Experimental setup: the Wien-Robinson bridge.

Equipment

Position No.	Material	Order No.	Quantity
1	Plug-in board,4mm plugs	06033-00	1
2	Resistor 500 Ohm 5%, 1W, G1	06057-50	1
3	Capacitor 10nF/ 250V, G1	39105-14	4
4	Resistor 1 kOhm, 1W, G1	39104-19	5
5	Short-circuit plug, white	06027-06	5
6	Difference amplifier	11444-93	1
7	PHYWE Digital Function Generator, USB	13654-99	1
8	30 MHz digital storage oscilloscope with colour display,2 x BNC cables l = 75 cm incl.	11462-99	1
9	Connector, T type, BNC	07542-21	1
10	Adapter, BNC-plug/socket 4 mm	07542-26	2
11	Connecting cord, 100 mm, yellow	07359-02	1
12	Connecting cord, 32 A, 500 mm, red	07361-01	2
13	Connecting cord, 32 A, 500 mm, blue	07361-04	3
14	Screened cable, BNC, l = 750 mm	07542-11	2
15	Screened cable, BNC, l 1500 mm	07542-12	2

Tasks

- Record the frequency response of the output voltage of
  1. a high-pass filter
  2. a low-pass filter
  3. a band-pass filter
  4. a Wien-Robinson bridge
  5. a parallel-T filter, point by point and to display the sweep on the oscilloscope.

- Investigate the step response of
  6. a differentiating network
  7. an integrating network

- Analyze and verify the measurements using the "measure" analysis software.
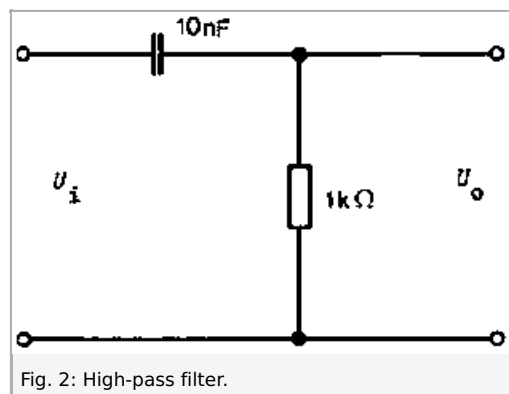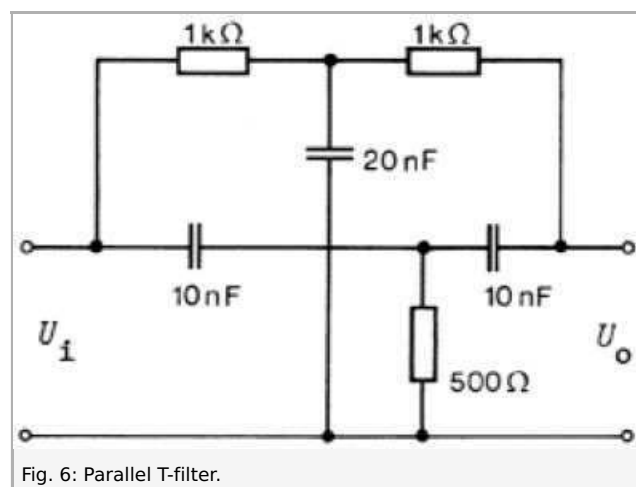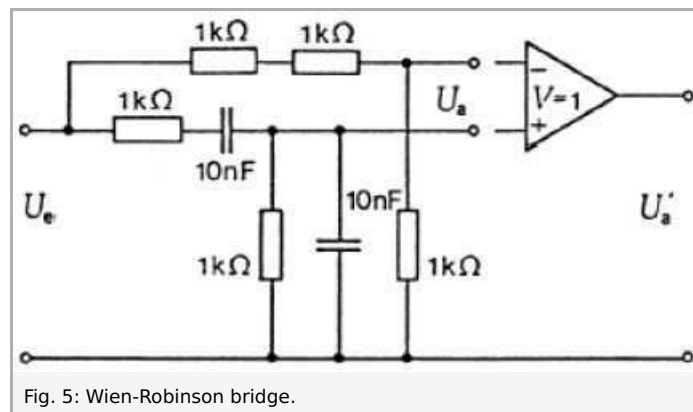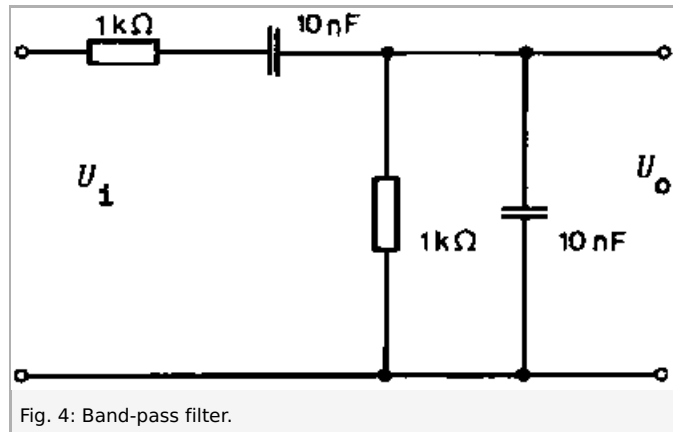
Set-up and procedure

Fig. 2: High-pass filter.

Fig. 3: Low-pass filter.

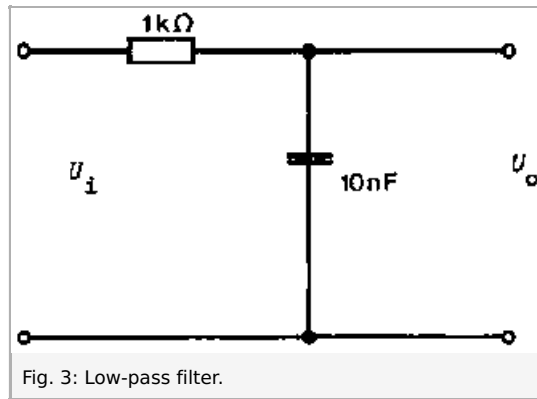Fig. 4: Band-pass filter.

Fig. 5: Wien-Robinson bridge.

Fig. 6: Parallel T-filter.

- Set up the experiment in accordance with Fig. 1 or the circuit diagrams in Figs. 2 to 6. To measure the frequency response point by point, the input and output voltages U_i and U_0 are measured with the oscilloscope. Split the generator signal with the BNC T-plug, as seen in Fig. 1. Connect one out-put directly to one of the oscilloscope channels, and the other through the analyzed circuit to the other oscilloscope channel. Adjust the oscilloscope channel settings to properly see

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both signals on the display.

- The frequency response can also be displayed on the oscilloscope by sweeping the generator frequency. In this case the U~f-output of the generator is connected to the X-input of the oscilloscope in X-Y operation (channel 1), while the signal passing through the circuit setup is connected to the Y-input of the oscilloscope (channel 2). Set the generator function to the $f_1 \ldots f_2$ mode and a logarith- mic range with adequate sweep factor ($f_n = f_{1-n}$*sweep factor). Enter the desired values for the fre- quency range (f_1 and f_2) and choose the $f_1 \ldots f_2$ mode from the U~f menu. The menu contains some preset frequency intervals which you can use to quickly change the frequency range you want to use. Set the display mode of the oscilloscope to persist-on and adjust the volts/div for each channel to obtain proper resolution of the signal on the screen. You will see that as the signal travels along the x-axis (hence frequency is increasing), the amplitude changes according to the filter characteristics. Example of input values for the high- and low-pass filters are: f_1 =1 kHz, f_2 =150 kHz, sweep = 1.03, oscilloscope channel 1 (U~f signal) Volts/div=1 Volt, oscilloscope channel 2 (filter signal) Volts/div=1 Volt.

- In the case of the Wien-Robinson bridge, the output voltage is not connected to earth although the generator and oscilloscope are and have the same earth potential, and we have to use the differen- tial amplifier (Note: there are several ways to set up the Wien-Robinson bridge with the provided el ements). It should be noted that the upper cutoff limit of the differential amplifier is 100 kHz when the input voltage is $U_i \leq 2V_{pp}$.

- The 20 nF capacitor required in the parallel-T filter is made up from two 10 nF capacitors connected in parallel.

- To investigate the square-wave behavior of the differentiating network (high-pass) and the integrat- ing network (low-pass), square-wave frequencies of 0.1, 1 and 10 times the cut-off frequency fc are fed to the input. Set the signal in the function generator to square-wave. Set the frequency to low (high) for the low-pass (high-pass) filter. Increase (decrease for high-pass filter) the signal frequency and observe the gradual change of the filter signal. The low-pass filter integrates high input frequen- cies (Fig. 12), while the high-pass filter differentiates low input frequencies (Fig. 11).

Theory and evaluation

1. The **high-pass filter** (Fig. 2) is a frequency-dependent voltage divider, whose behavior is described in complex terms by

$$A\left(iw\right)\frac{U_0}{U_i} = \frac{R}{R + \frac{1}{iwC}} = \frac{1}{1 + \frac{1}{iwRC}} \tag{1}$$

where $i = \sqrt{-1}$ is the imaginary unit, ω is the frequency, U_0 and U_i are the output and input voltage, respectively, R is the resistance and C is the capacitance. From eq. (1) it follows that the amplitude-frequency dependence is given by

$$|A| = \frac{1}{\sqrt{1 + \frac{1}{w^2 R^2 C^2}}} \tag{2}$$

The 3 dB cut-off frequency f_c is the frequency at which

$$|A| = \frac{|U_0|}{|U_i|} = \frac{1}{\sqrt{2}} \tag{3}$$

By substituting (3) into (2) we obtain

$$f_c = \frac{w_c}{2\pi} = \frac{1}{2\pi RC} \tag{4}$$

2. The **low-pass filter** (Fig. 3) is characterized by the following behavior

$$A\left(iw\right) = \frac{U_0}{U_i} = \frac{\frac{1}{iwC}}{R + \frac{1}{iwC}} = \frac{1}{1 + iwRC} \tag{5}$$

Hence the dependence of the ratio of the signal amplitudes on frequency is given by eq. (6)

$$|A| = \frac{1}{\sqrt{1 + w^2 R^2 C^2}} \tag{6}$$

and the 3 dB cut-off frequency f_c is

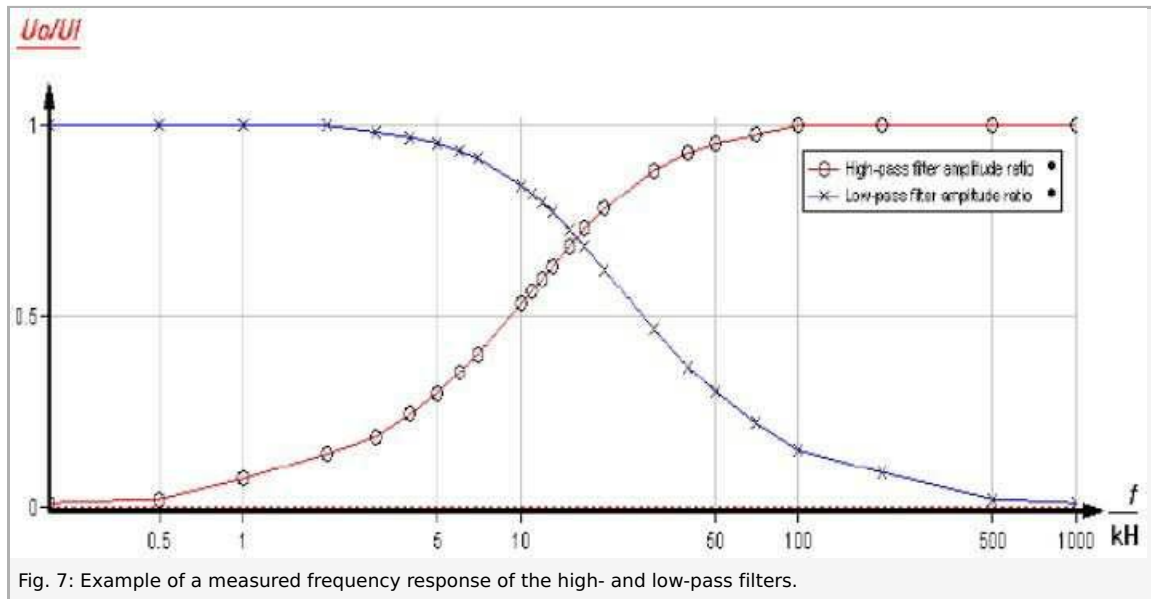$$f_c = \frac{w_c}{2\pi} = \frac{1}{2\pi RC} \tag{7}$$

Fig. 7: Example of a measured frequency response of the high- and low-pass filters.

3. If we connect a high-pass and a low-pass filters together, we obtain a **band-pass filter** (Fig. 4). The following equation, with the abbreviation $wRC = \Omega$, applies to the unloaded circuit:

$$\underline{A}\left(iw\right) = \frac{U_0}{U_i} = \frac{\frac{1}{\frac{1}{R}+iwC}}{\frac{1}{\frac{1}{R}+iwC}+R+\frac{1}{iwC}} = \frac{i\Omega}{1+3i\Omega-\Omega^2} \tag{8}$$

Hence we obtain for the amplitude

$$|\underline{A}| = \frac{1}{\sqrt{9+(\Omega-\frac{1}{\Omega})^2}} \tag{9}$$

The output voltage has its maximum value for $\Omega = 1$, hence the mid-band frequency is

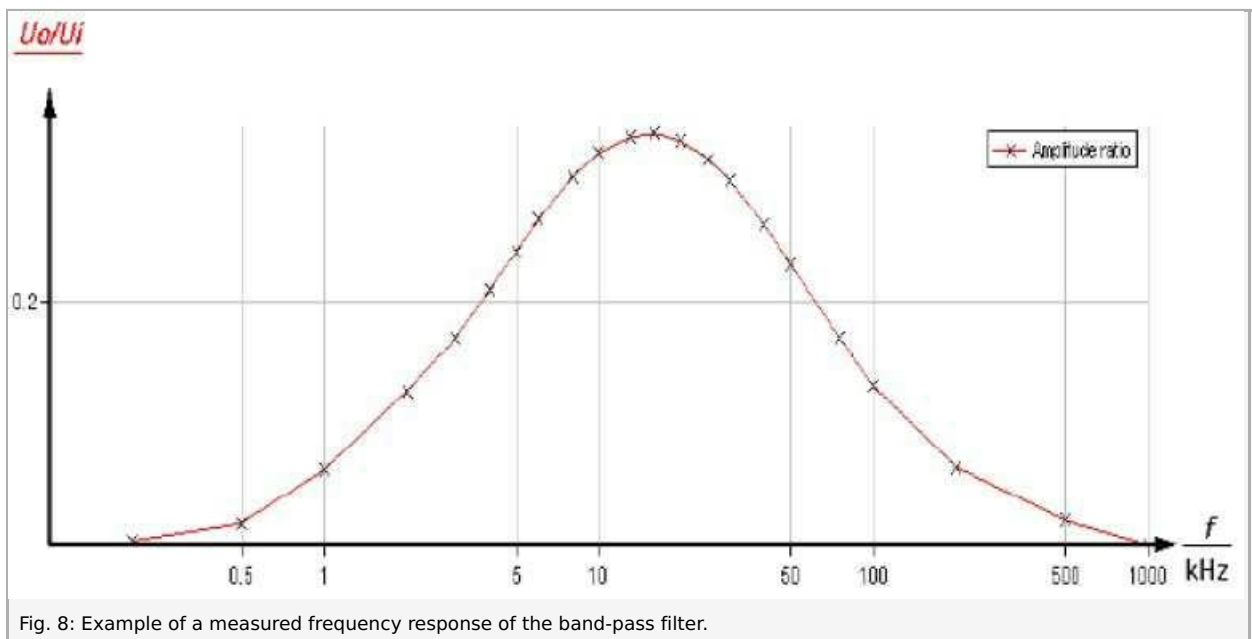$$f_m = \frac{w_m}{2\pi} = \frac{1}{2\pi RC} \tag{10}$$

Fig. 8: Example of a measured frequency response of the band-pass filter.

4. If we add a parallel-connected voltage divider consisting of ohmic resistors in the ratio 2 :1 to the band-pass filter, we obtain a **Wien-Robinson bridge** (Fig. 5):

$$\underline{A}\left(iw\right) = \frac{U_0}{U_i} = \frac{1}{3} - \frac{i\Omega}{1+3i\Omega-\Omega^2} \tag{11}$$

which gives

$$|\underline{A}| = \frac{1-\Omega^2}{3\sqrt{9\Omega^2 + (1-\Omega^2)^2}}$$

(12)

The output voltage has its minimum value at

$$f_m = \frac{w_m}{2\pi} = \frac{1}{2\pi RC}$$
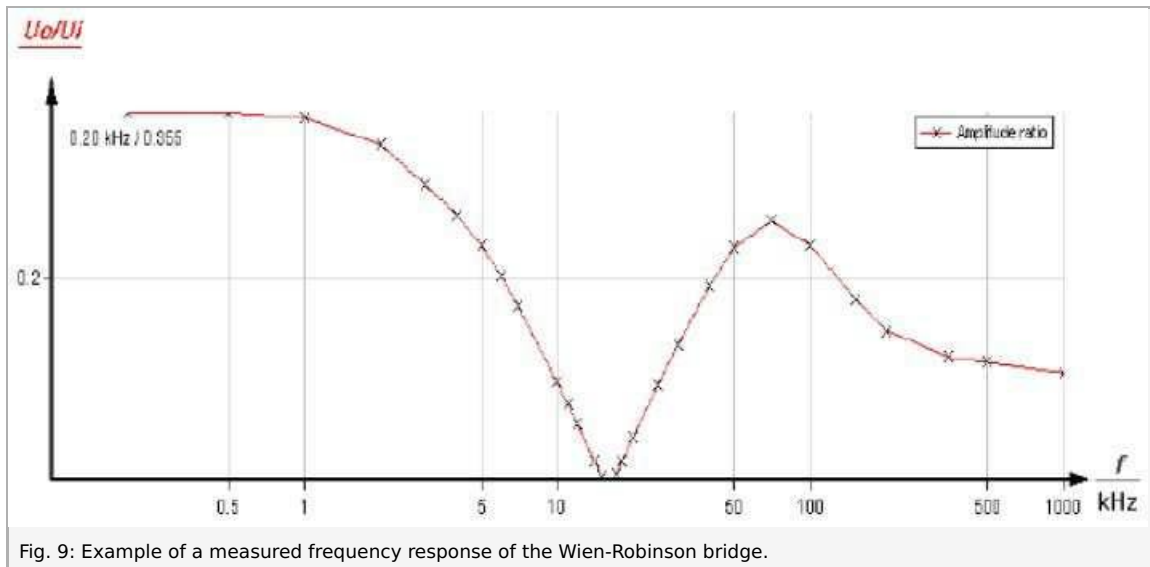
(13)

Fig. 9: Example of a measured frequency response of the Wien-Robinson bridge.

5. Unlike the Wien-Robinson bridge, the **parallel-T filter** (Fig. 6) has the advantage that the output vol-tage is connected to earth. Using the nodal equation we obtain

$$\underline{A}(iw) = \frac{U_0}{U_i} = \frac{i\Omega}{1+4i\Omega-\Omega^2}$$

(14)

Hence

$$|\underline{A}| = \frac{1-\Omega^2}{\sqrt{16\Omega^2 + (1-\Omega^2)^2}}$$

(15)

The output voltage has its minimum value at

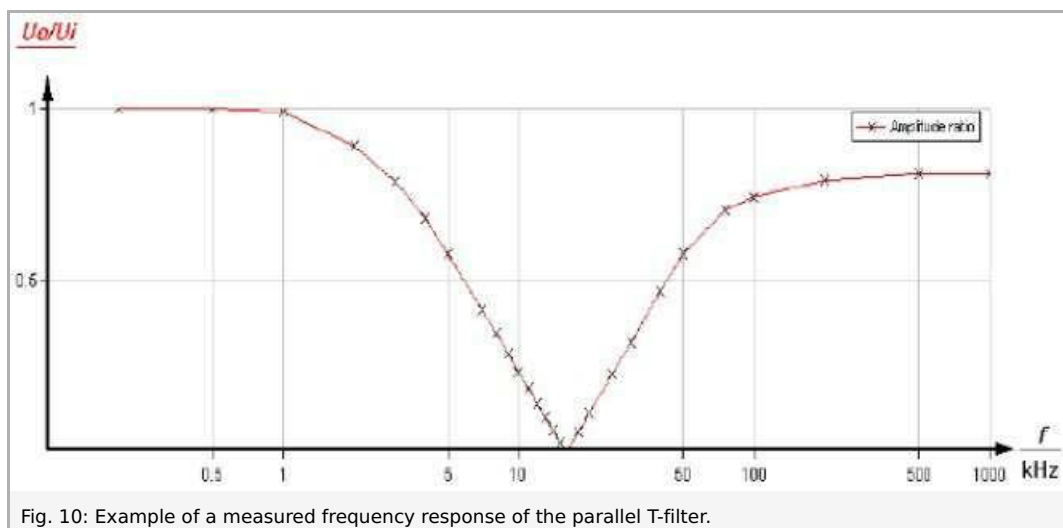$$f_m = \frac{w_m}{2\pi} = \frac{1}{2\pi RC}$$

(16)

Fig. 10: Example of a measured frequency response of the parallel T-filter.

6. Investigation of the **step response of a differentiating network**.

$$C\frac{d}{dt}(U_i - U_0) - \frac{U_0}{R} = 0 \tag{17}$$

is applied to the unloaded output to calculate the step response of the differentiating network (high-pass). If input voltages U_i are used with frequencies $f \ll f_c$ then $|U_0| \ll |U_i|$ and it follows from eq. (17) that

$$U_0 = RC\frac{d}{dt}U_i \tag{18}$$

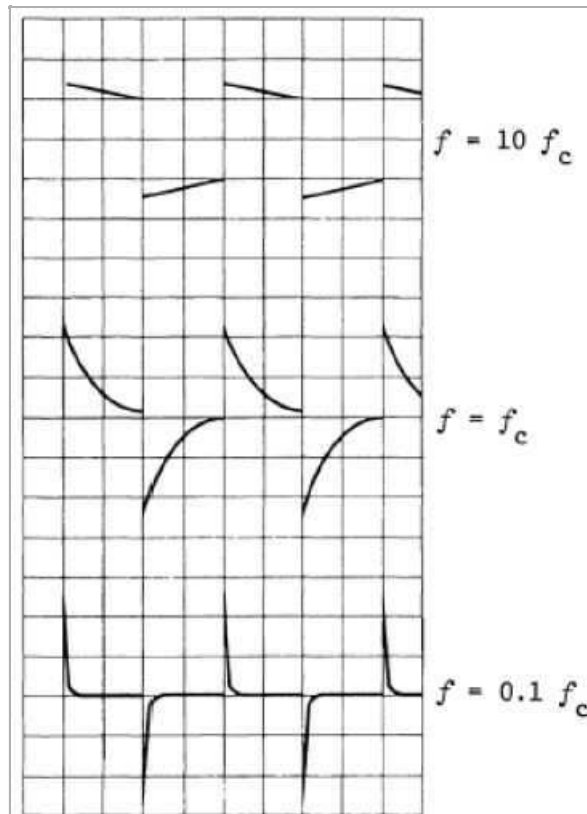Low-frequency input voltages are therefore differentiated (Fig. 11).

Fig. 11: Square-wave behavior of the high-pass filter at various frequencies (low input voltages are differentiated).

7. Investigation of the **step response of an inte-grating network**.
   Applying the nodal equation to the unloaded output of the low-pass filter gives

$$\frac{U_i - U_0}{R} - C\frac{d}{dt}U_0 = 0$$

If input voltages U_i are used with frequencies $f \gg f_c$ then $|U_0| \ll |U_i|$ and from eq. (19) we obtain

$$\frac{d}{dt}U_0 = \frac{1}{RC}U_i \tag{20}$$

therefore

$$U_0 = \frac{1}{RC}\int U_i\,dt \tag{21}$$

Hence, high input voltages are integrated (Fig. 12).

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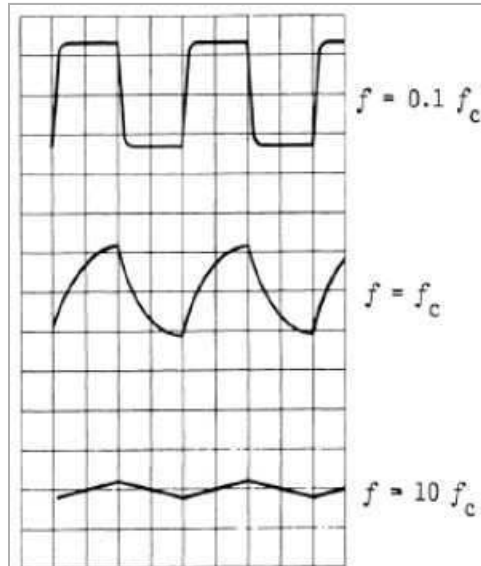Fax: +49 551 604 - 107

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Fig. 12: Square-wave behavior of the low-pass filter at various frequencies (high input voltages are integrated).

Data analysis

The measurement results can be plotted and analyzed with the "measure" software. To enter the data manually, choose Enter data manually from the Measurement menu (Fig. 13).
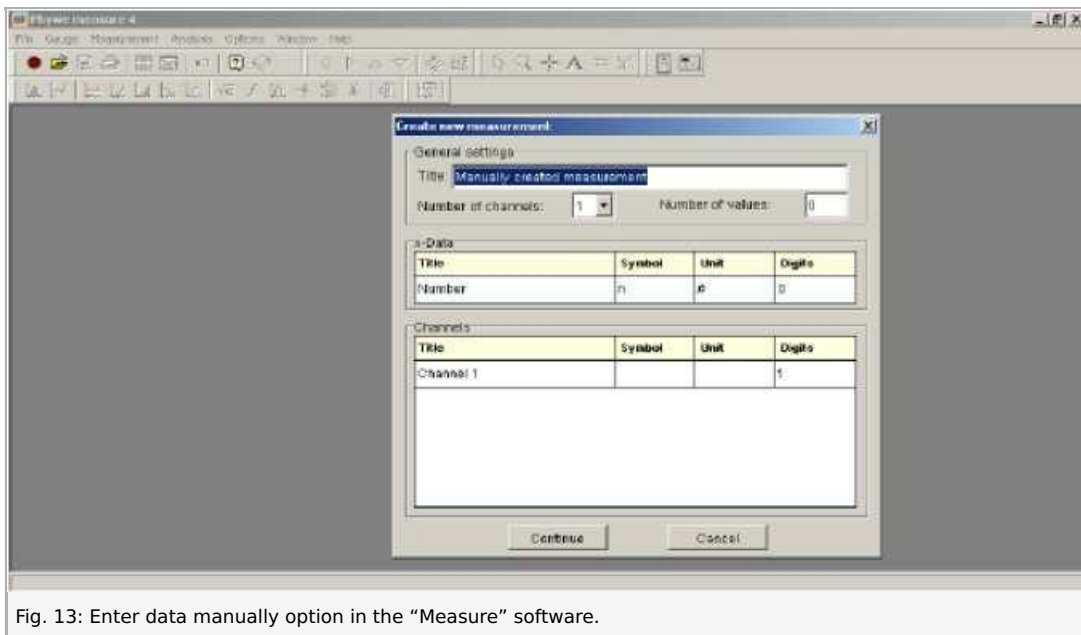

Fig. 13: Enter data manually option in the "Measure" software.

Choose the adequate number of channels (for example, if you measured amplitude vs. frequency, choose 2 channels, if you want to plot the frequency response for two filters on the same plot, like Fig. 7, choose 3 channels, etc.). The x-data field creates integer numbers of the specified length (number of values) starting with 1. Enter the number of values that corresponds to the number of measured data points, and enter the Title, Symbol, Unit and Digits for each channel. Note: you can adjust these values later, as well as specify which channel is plotted on the x and y axes. When done, click Continue.

A table opens, where you can input the measurement values for each channel (Fig. 14). You can add and remove measurement points using the symbols at the bottom of the table. Click OK when done. A plot of the data appears.
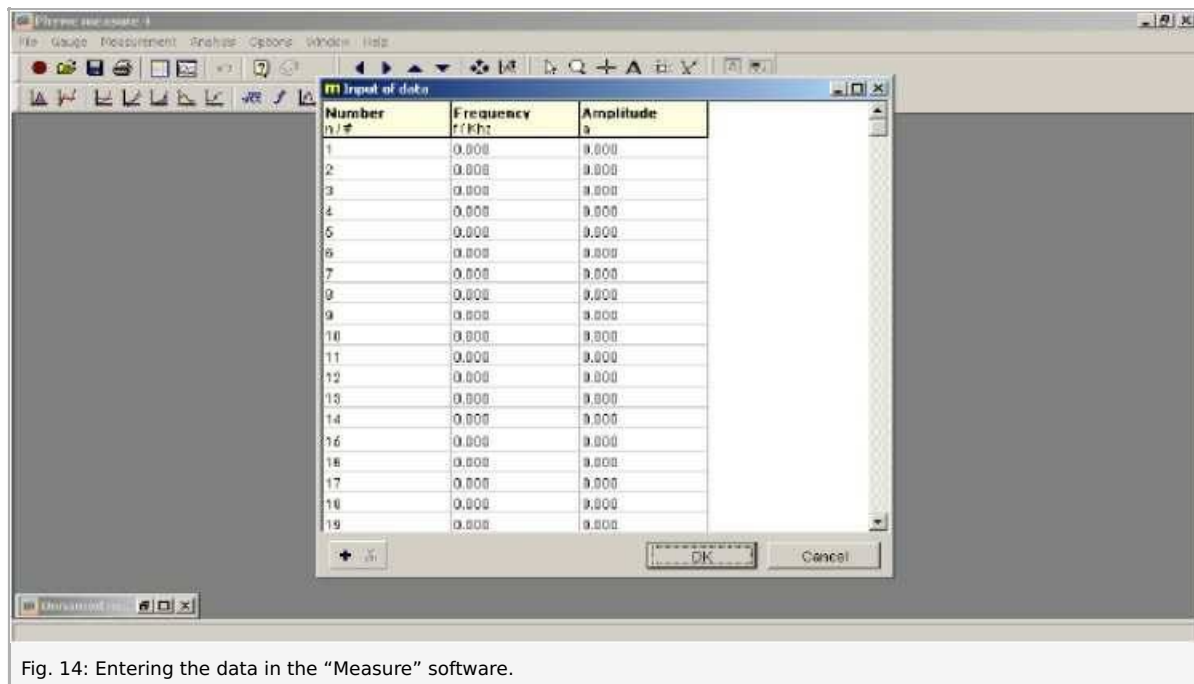
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Fig. 14: Entering the data in the "Measure" software.

Go to Channel manager in Measurement menu to specify the data channels to be plotted as the x and y axes. In the left column you can see the data channels you entered. The right column will be the new ar-rangement of the x and y axes. Notice the field highlighted in blue in the right column (default is the Des-tination x-axis field, but you can change that by double-clicking on a required field). Mark a data channel in the left column, and then click on the transfer arrow between columns. The channel now appears in the highlighted field in the right column. Hence, to create plots of frequency vs. amplitude ratio, choose frequency as the new x-axis, and amplitude as the y-axis. Click OK when done.

You can easily access the data table and display options by clicking on the plot with the right mouse but-ton. For example, to obtain a plot like Fig. 7 from the frequency and amplitude data, go to display op-tions, choose the X-data tab, and change scaling to logarithmic. You can adjust other display options, like the x and y ranges and labels etc.

It is recommended you save the measurement after you enter the data, you can do that at any time from the File menu. You can load saved data with the Open experiment command in the File menu. Note: many commands are also accessible directly through corresponding icons located in the taskbar.

To analyze the plotted data, scroll down the Analysis menu and choose the required option, for example Show extrema. A window appears, showing the extrema of the plot (Fig. 15). Mark the Visualize results option to display the values on the plot. You can export a plot to a file using the Export data option from the Measurement menu. Mark Save to file and Export as bitmap to save the plot as a bitmap image.
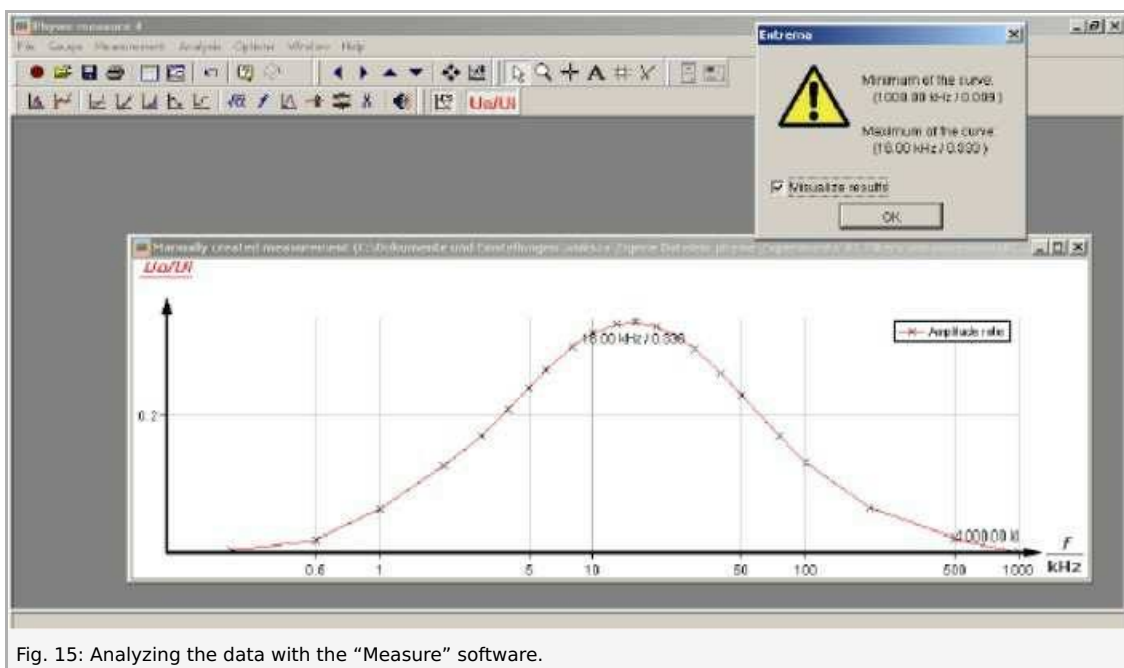

Fig. 15: Analyzing the data with the "Measure" software.

You can also use the "measure" software to generate and modify theoretical data, for example, to check what the differential and integral of a square-wave function look like. Go to the Measurement menu and choose Function generator (Fig. 16). Choose the square-wave function and set the corresponding para-meters (you can use those seen in Fig. 16). Click Calculate. A plot of

the square-wave function appears. Note: you can generate any function by clicking the Equation tab and entering a formula. Now choose Channel modification from the Analysis menu. Mark differentiate (integrate) and click Calcu-late. The differential (integral) of the signal is added onto the plot.
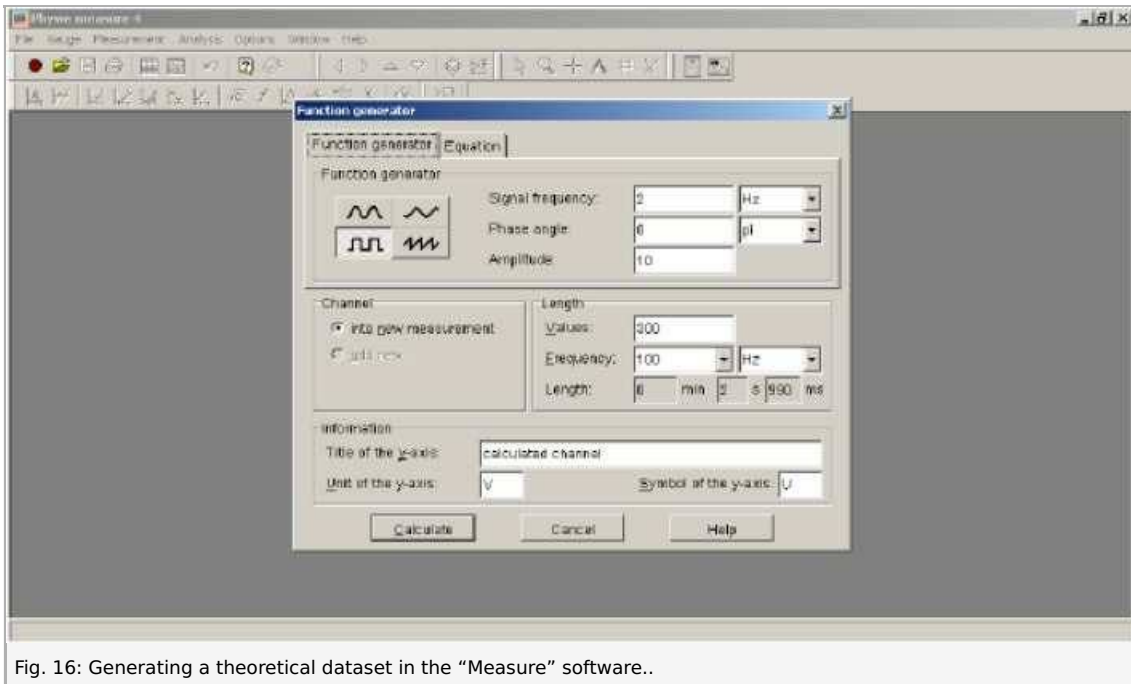

Fig. 16: Generating a theoretical dataset in the "Measure" software..