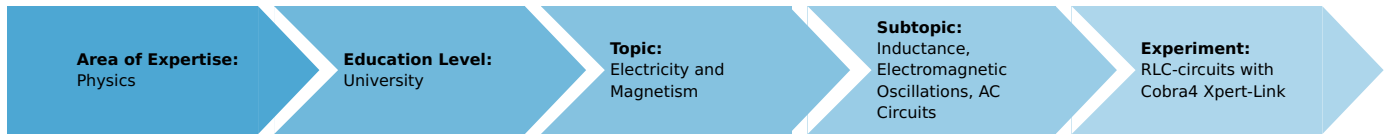


RLC-circuits with Cobra4 Xpert-Link (Item No.: P2440664)

Curricular Relevance



Difficulty



Very difficult

Preparation Time



20 Minutes

Execution Time



20 Minutes

Recommended Group Size



2 Students

Additional Requirements:

Experiment Variations:

Keywords:

Damped and forced oscillations, resistance, capacitance, inductance, reactance, impedance, phase displacement, Q-factor, band-width

Introduction

Overview

The resonance-behaviour of an RLC-circuit is studied and the resonance frequencies f_{res} are determined and compared with the theoretical values.

$$f_{res} = \frac{1}{2\pi\sqrt{LC}}$$

The resonance curves are measured and the impedance-behaviour of the LC-component is analysed. Further the bandwidths B and the quality factors Q are determined from the resonance curves and compared with the theoretical values for a series-tuned circuit, obtained from the parameters of the electrical components.

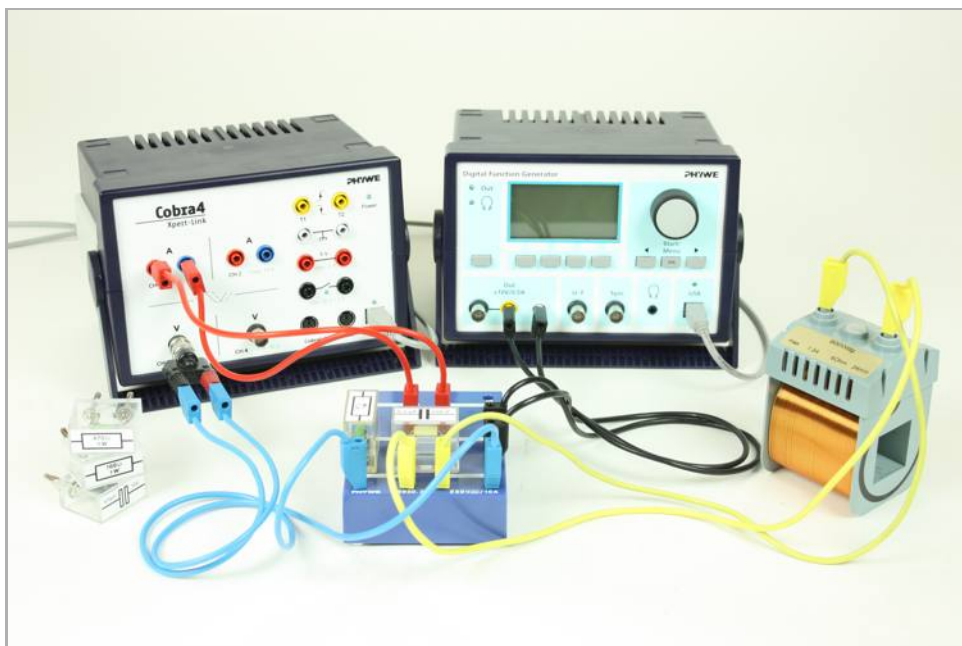


Fig. 1: Experimental setup (parallel connection).

Equipment

Position No.	Material	Order No.	Quantity
1	Cobra4 Xpert-Link	12625-99	1
2	PHYWE Digital Function Generator, USB, incl. Cobra4 software	13654-99	1
3	Coil, 900 turns	06512-01	1
4	Cobra4 Xpert-Link set of cables	12625-10	1
5	Connection box	06030-23	1
6	Capacitor 470nF/250V, G1	39105-20	1
7	Capacitor 100 nF/250V, G1	39105-18	1
8	Resistor 100 Ohm, 1W, G1	39104-63	1
9	Resistor 47 Ohm, 1W, G1	39104-62	1
10	Resistor 470 Ohm, 1W, G1	39104-15	1
11	Software measureLAB	14550-61	
	PC, Windows® 7 or higher		

Tasks

1. Measure the voltage drop U over the LC-component and the current I through the circuit and determine the resonance frequency for both combinations of coil and capacitor
2. Determine the impedance Z of the LC-component for both circuits
3. Determine the bandwidth B and Q-factor for both circuits

Set-up and procedure

The experimental set-up for measuring the voltage and current in the series-tuned circuit is shown in Fig. 2. R_i denotes the internal resistance of the digital function generator, which is given in the technical description as $R_i = 2\ \Omega$.

The experimental set-up for measuring the voltage and current in the parallel-tuned circuit is shown in Fig. 3.

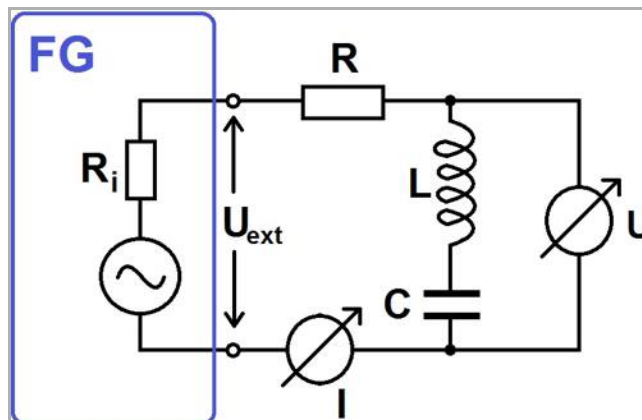


Fig. 2: Circuit diagram (series circuit).

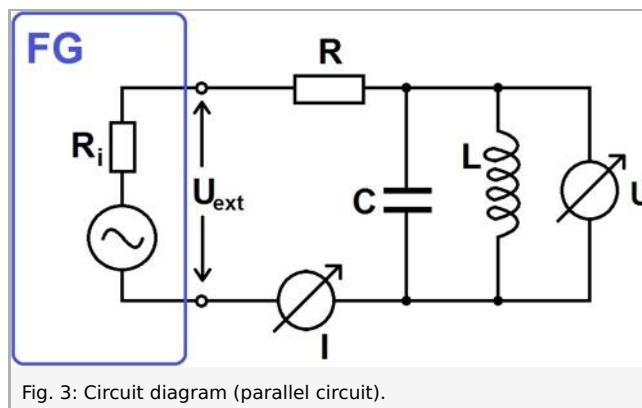


Fig. 3: Circuit diagram (parallel circuit).

In the beginning, set up the equipment according to the series circuit diagram. Start with a combination of a $47\ \mu\text{F}$ capacitor and a $470\ \Omega$ resistor. Use the Xpert-Link CH1 I1 channel for the measurement of the current and the CH3 U1 channel for voltage measurement.

Connect both the Xpert-Link and the function generator to your computer.

Start the software "measureLAB", and choose the experiment from the start screen (choose "PHYWE experiments", search for "P2440664", and click on the folders that contain this experiment). All necessary presettings will be loaded.

If you wish to define your own settings, click on the gear wheel button, choose "Sensors/Channels" and select "Xpert-Link" Channel CH1. The measurement range has to be set to 10 mA, and in order to obtain effective values, "TRMS" needs to be selected. CH3 has to be set to 10 V and TRMS active as well. The function generator settings can be modified when choosing "Sensors/Channels" and "Function generator". Use the settings that are provided in the settings overview below. For the first experiment, a frequency ramp in the range 1 200 Hz - 2 000 Hz is suitable (increment: 5 Hz). A sampling rate of 1 kHz will be sufficient. When loading experiment presettings you can start without further modification. The frequency ramp can be narrowed down for different capacitors. Refer to the example measurements for the definition of suitable boundaries (click on the function generator symbol and the gear-wheel button).

Settings overview (first experiment, $C = 0.47\ \mu\text{F}$, $R = 470\ \Omega$):

CH1: 10 mA, Average 1 value, TRMS active

CH3: 10 V, Average 1 value, TRMS active

Function generator (Power output): Frequency ramp 1 200 Hz-2 000 Hz, Increment 5 Hz, Pause time 200 ms

Output form: linear

Signal form: Sinusoidal, Amplitude pp. 5 V, Offset 0 V

Sampling rate: 1 kHz

Before performing the experiment, it is important to understand the way the TRMS measurement works, as the digital function generator settings have to be adjusted accordingly. A TRMS converter that transforms the signal to effective values requires a certain time for this operation in order to give reliable values. The converter will only give precise TRMS values if this period of time has passed until the signal is modulated again! In the case of the Xpert-Link, the conversion time is 200 ms. This is why the

pause time of the frequency ramp should not be below this value! Also, the smaller the change in the input signal, the more correct will be your TRMS data. This means that when recording the resonance curves you need to decrease the frequency ramp increment, as this quantity accounts for the variation of the signal (choosing larger values for the increment will result in “steps” in your resonance curves). It is recommended to choose a larger increment (5 Hz or 10 Hz) for an overview scan, and choosing a smaller step size (1 Hz or 2 Hz) when investigating a narrow region of interest. Please note that measurement using a smaller step size (increment) may be considerably more time-consuming (depending on the PC hardware that you are using).

Also note that the Xpert-Link current measurement range has to be increased to 100 mA when using smaller resistances, as it is the resistor that will limit the current in the circuit.

For a live measurement of the impedance virtual channels are used. When loading the presets, a predefined virtual channel “Impedance” will measure the ratio CH3/CH1 as a function of the frequency. For measurements including the current and voltage channel click on the Xpert-Link symbol in the devices bar, and drag and drop the desired channels into the diagram. If you wish to switch to a new diagram containing only the impedance as a function of frequency, simply reload the provided presets.

If you wish to define own virtual channels, click on the $\sqrt{\alpha}$ button in the devices bar, then click on the “+” button, and the virtual channel settings menu will open. This menu allows for the definition of new virtual channels by giving values for number of decimal places, unit, channel name and symbol, and by giving the formula that defines the way the channel will be calculated. Click on the channel symbols given below in order to enter these channels into the formula, or drag and drop them into the formula input mask. You can use arithmetic operations and trigonometric functions for the definition of virtual channels by entering the desired operations with your keyboard. If an error in the formula occurs, e.g. brackets are not closed, a red frame will indicate that you need to correct the entry before performing the measurement.

During the experiment, you may wish to remove previous measurements from the diagram. To do so, click on the “x” symbol next to the measurement name in the diagram caption, or delete all of the measurements from the diagram by clicking on the “x” symbol in the analysis tools tab in the diagram. You can export your data by clicking on the “Data pool” symbol, and download your measurements from the data pool by clicking on the download symbol. Now perform measurements for the different types of networks, as described below.

If you accidentally closed the diagram, you can either reload the predefined settings, or click on the virtual channel or Xpert-Link channels in the devices bar, and then click on “diagram”, or drag and drop the diagram onto the workspace. The second channel can be added by drag and drop into the diagram, and the frequency can be selected as the channel that defines the x-axis by choosing “frequency” from the channel list in the drop down menu of the diagram representation tab in the diagram. **For the very first measurement, add current CH1 and voltage CH3 by drag and drop into the diagram.**

Start the measurement by clicking the “Record” button. The measurement will stop automatically as a frequency ramp is defined. Your results may look like this:

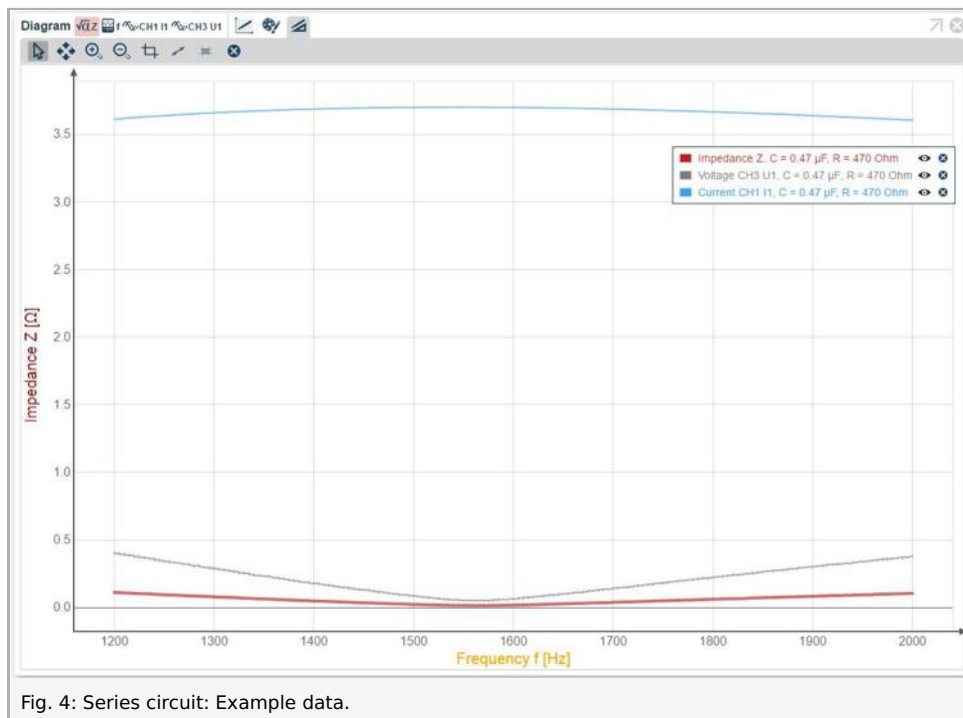


Fig. 4: Series circuit: Example data.

You may wish to use a logarithmic display of your data. To do so, click on the „display” tab in the diagram and choose „Log” for the y-axis.

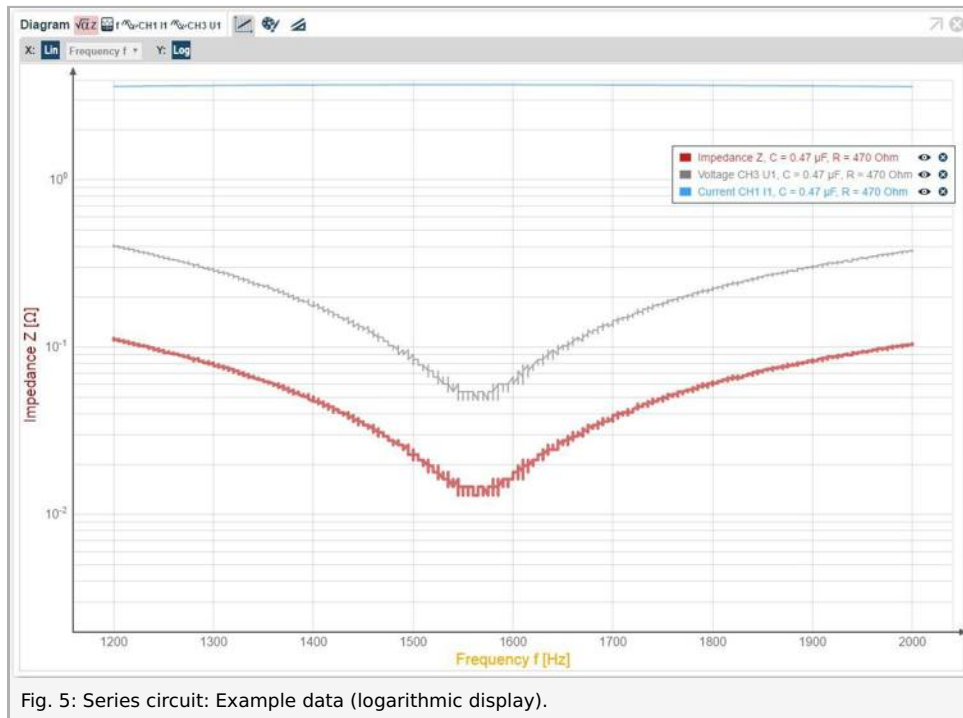


Fig. 5: Series circuit: Example data (logarithmic display).

Now choose a specific combination of capacitance and resistance, and measure the current as a function of frequency. You will need this data for bandwidth evaluation. Your result may look like this:

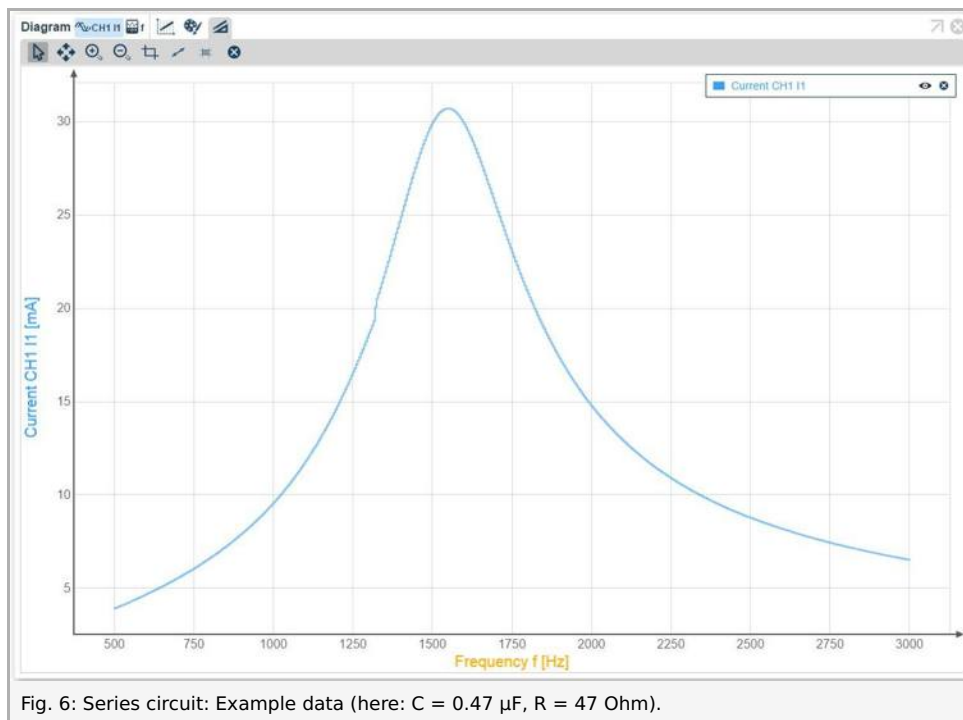


Fig. 6: Series circuit: Example data (here: C = 0.47 μF, R = 47 Ohm).

Use this measurement to determine both resonance frequency and bandwidth (survey tool) from the current signal. In this example data, a bandwidth of 428 Hz corresponds to a resonance frequency of 1 550 Hz (see Fig. 7).

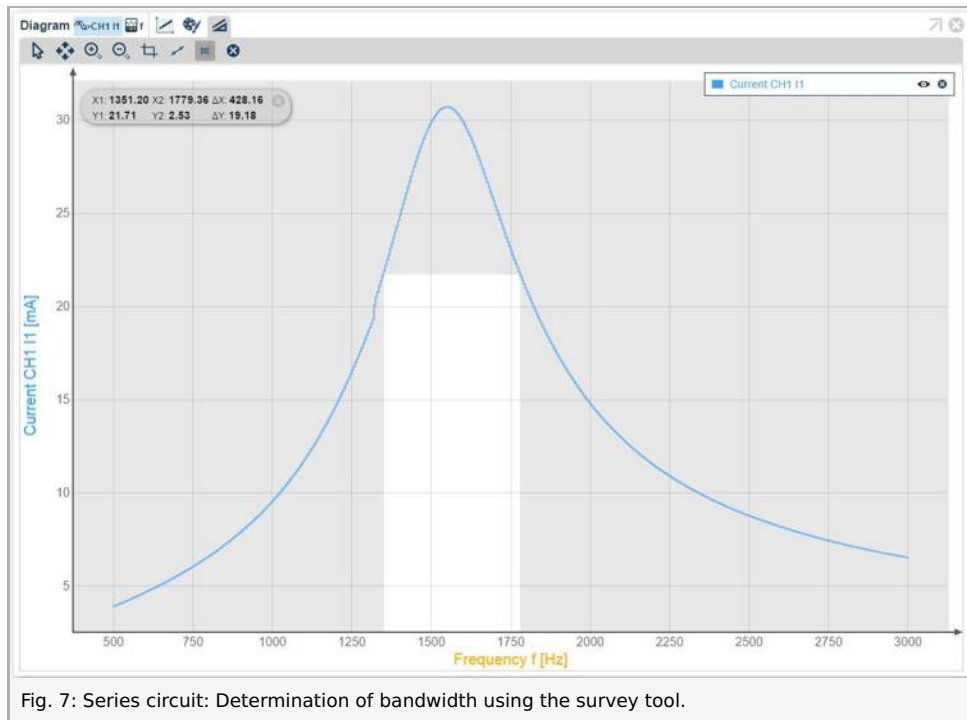


Fig. 7: Series circuit: Determination of bandwidth using the survey tool.

Now switch the capacitor and repeat the impedance measurement for all the capacitors that are provided with this experiment. You may wish reload the provided presettings, as the current and voltage channels are not needed for the next measurement. You will need to adjust the frequency ramp to the respective resonance frequencies, as an overall frequency ramp will be rather time-consuming (depending on your PC hardware configuration). Choose an increment of 5 Hz for a coarse overview, and refer to Fig. 8 for an estimate for the frequency ramp range. You may wish to repeat your measurements at the positions of the resonance peaks derived from this overview, and decrease the increment for a more detailed scan of a narrower region of interest, thus providing more precise data for the determination of the resonance frequencies.

Your result may look like depicted in Fig. 8 and Fig. 9.

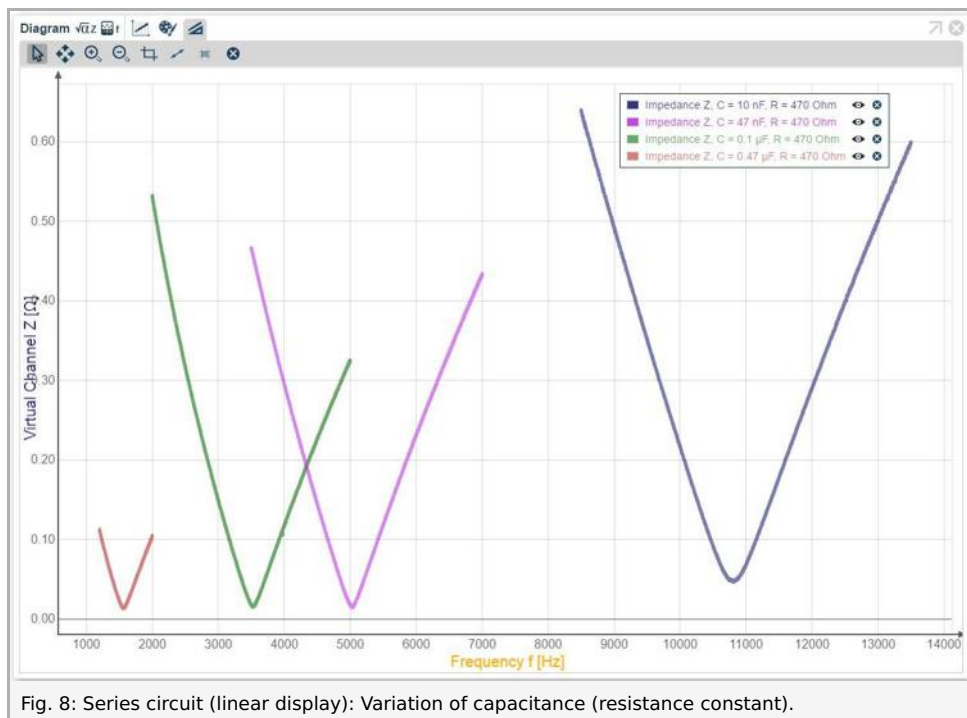


Fig. 8: Series circuit (linear display): Variation of capacitance (resistance constant).

Note that the position of the peak shifts for different capacitances. Capacitors are frequency-dependent hardware components! If you choose the same resistance in both the series-circuit and the parallel-circuit experiment, you can directly compare resonance frequencies for both experiments. For further clarification, you may wish to use a logarithmic display of your data. Now your results may look like this:

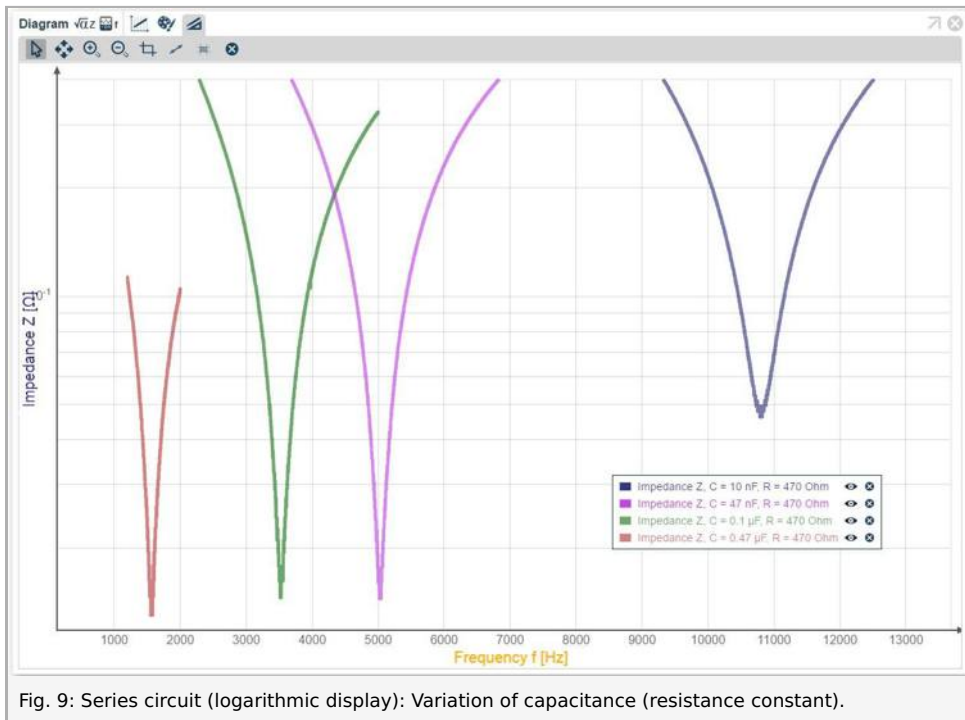


Fig. 9: Series circuit (logarithmic display): Variation of capacitance (resistance constant).

Point the mouse cursor to your data in order to determine the exact values for the respective resonance frequencies. You can use the mouse scroll wheel to zoom into your data.

Now change your experiment to the parallel-tuned circuit. Start with a configuration of $C = 0.47 \mu\text{F}$ and $R = 470 \Omega$, or any other combination of components that you wish to investigate.

Add the current and voltage channels to your diagram, choose a corresponding frequency ramp and a suitable increment. Your result may look like this:

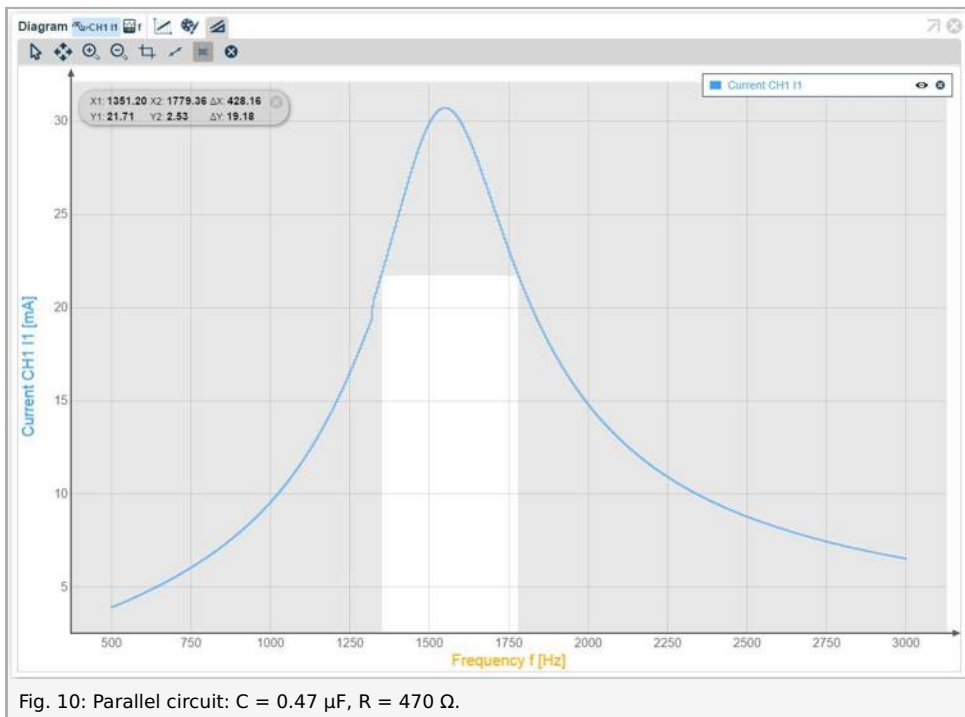


Fig. 10: Parallel circuit: $C = 0.47 \mu\text{F}$, $R = 470 \Omega$.

Use this measurement to determine both resonance frequency and bandwidth (survey tool) from the voltage signal. In this example data, a bandwidth of 605 Hz corresponds to a resonance frequency of 1 588 Hz (see Fig. 11).

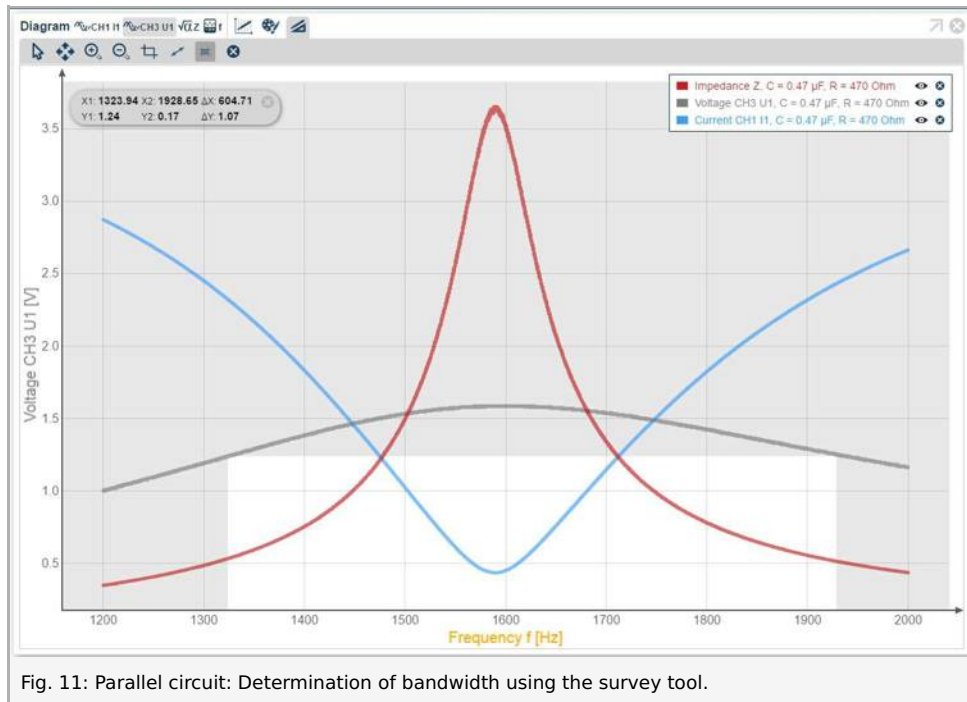


Fig. 11: Parallel circuit: Determination of bandwidth using the survey tool.

Now switch the capacitor and repeat the measurement for all the capacitors that are provided with this experiment. Your results may look like this:

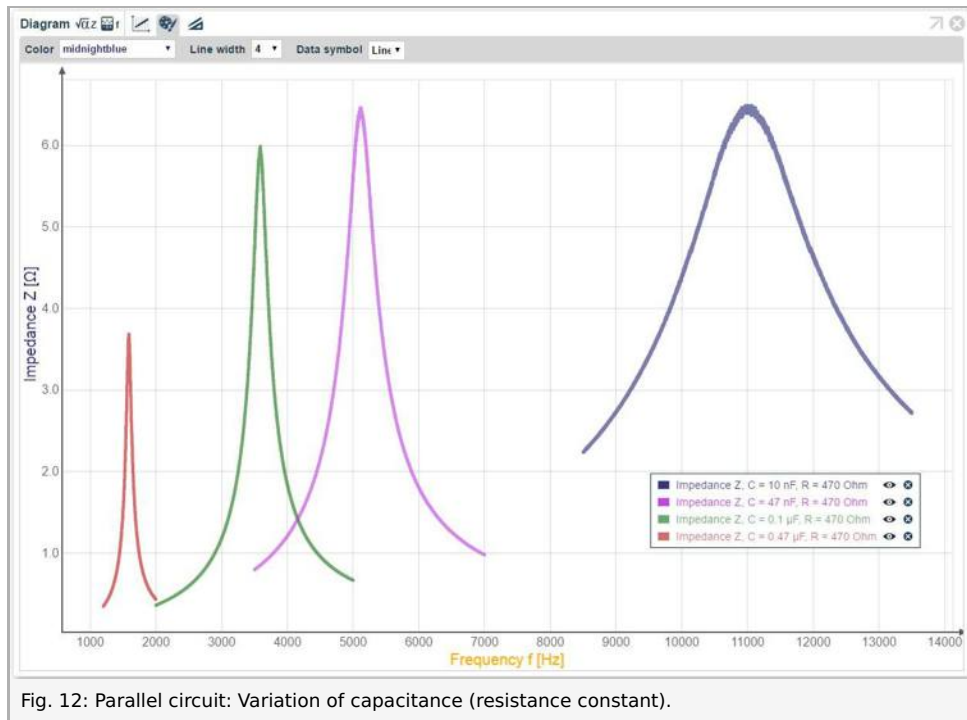


Fig. 12: Parallel circuit: Variation of capacitance (resistance constant).

Note that the position of the peak shifts for different capacitances. If you choose the same resistance in both the series-circuit and the parallel-circuit experiment, you can directly compare resonance frequencies for both experiments.

In the next step choose a capacitor that will be kept constant throughout the next experiment, and repeat the measurement switching all the resistors that are provided with this experiment. Your results may look like this:

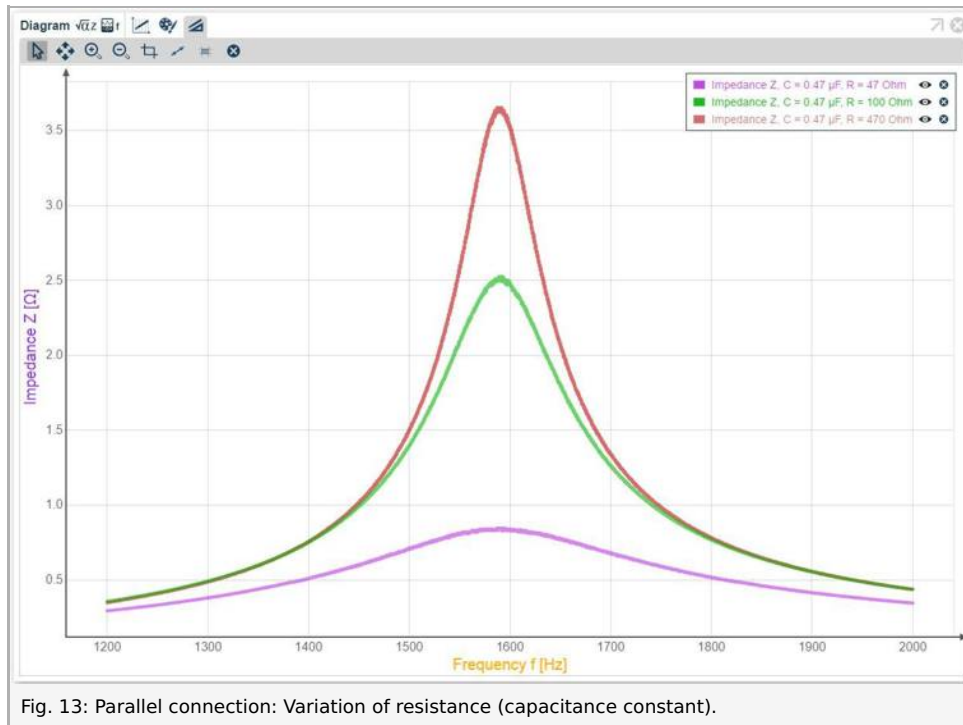


Fig. 13: Parallel connection: Variation of resistance (capacitance constant).

Note that the position of the peak is independent from the resistance. Resistors are frequency-independent hardware components! (You will find the same result when performing the resistance variation using the series-tuned circuit.)

You may also wish to investigate the influence of the coil (number of turns) by using coils with different numbers of turns that may be available in your lab.

Theory and evaluation

Theory

An RLC-circuit (also oscillating, oscillator or resonant circuit) consists of a resistor (R), an inductance (L) and a capacitor (C) – sometimes it is also referred to as LC-circuit, because the resistor is used to simulate the loss-resistance of a real circuit. Generally one differs between two kinds of RLC-circuits, the series- and the parallel-tuned circuit. The corresponding circuit diagrams are shown below.

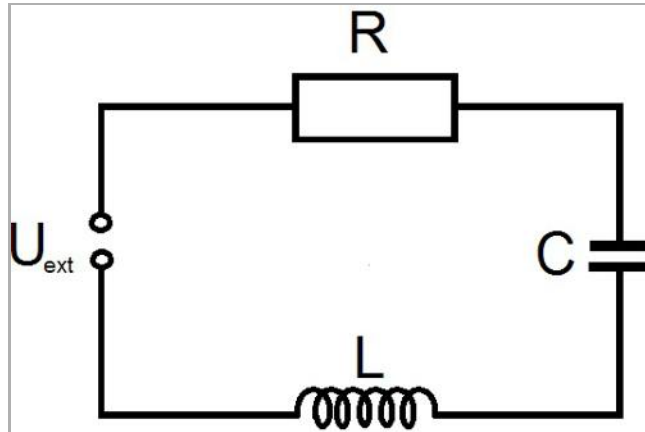


Fig. 14: Circuit diagram for a series-tuned RLC-circuit

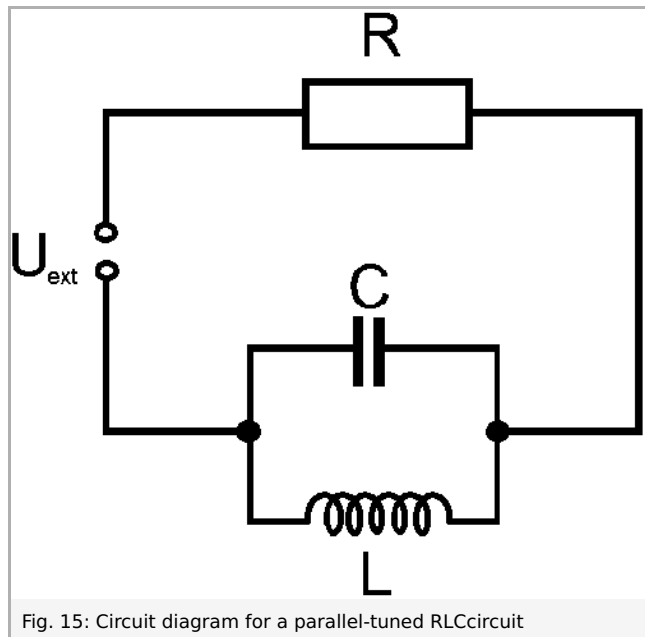


Fig. 15: Circuit diagram for a parallel-tuned RLC-circuit

When a fully charged capacitor is discharged through an inductance coil, the discharge current induces a magnetic field in the coil, which reaches its maximum, when the capacitor is completely discharged. Then, due to the decreasing current, the change in the magnetic field induces a voltage which according to Lenz's law charges the capacitor. Now the current decreases to zero until the capacitor is completely charged again, but with reversed sign of charges. At this point, the procedure starts again, but with opposite direction of the current. In absence of any resistance, this charging and discharging would oscillate forever – but because of ohmic resistances which every real circuit possesses, the oscillation is damped and so the amplitude of current and voltage decreases by time.

According to Kirchoff's law the total voltage in one loop must add to zero or be equal to an external potential. Therefore we obtain for the circuit in Fig. 1:

$$U_L + U_C + U_R = U_{ext} \tag{1}$$

where

-

$$U_L = L \frac{d}{dt} I \quad \text{is the voltage drop across the inductance } L \quad (2.1)$$

•

$$U_C = \frac{Q}{C} \quad \text{is the voltage drop across the capacitor } C \quad (2.2)$$

•

$$U_R = RI \quad \text{is the voltage drop across the resistor } R \quad (2.3)$$

•

$$U_{\text{ext}} = U_{\text{FG}} = U_0 \cdot e^{i\omega t} \quad \text{is the external voltage, which in our case is the output of the function generator} \quad (2.4)$$

Using these identities and differentiating (1) with respect to time t , one obtains with $\frac{d}{dt} Q = I$

$$L \frac{d^2}{dt^2} I + R \frac{d}{dt} I + \frac{1}{C} I = i\omega U_0 \cdot e^{i\omega t} \quad (3)$$

This equation can be easily transformed into the inhomogeneous differential equation for the forced oscillation; by using Euler's formula, $\omega_0 = \frac{1}{\sqrt{LC}}$ and the damping coefficient $\delta = \frac{R}{2L}$ one obtains

$$\ddot{I} + 2\delta \dot{I} + \omega_0^2 I = \frac{\omega}{L} U_0 \cdot e^{i(\omega t + \pi/2)} \quad (4)$$

The real part of the solution for (3) gives the current

$$I = I_0 \cdot \cos(\omega t - \varphi) \quad (5)$$

with

$$I_0 = \frac{U_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad (6)$$

The phase displacement ϕ is given by

$$\tan \phi = -\frac{1}{R} \left(\omega L - \frac{1}{\omega C} \right) \quad (7)$$

and the resonance point is found at

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}} \quad (8)$$

The impedance (value) is defined by $Z = \frac{U_{\text{eff}}}{I_{\text{eff}}}$. From (6) one obtains for the LC-component of the series-tuned circuit

$$Z_s = \left| \omega L - \frac{1}{\omega C} \right| \quad (9)$$

(the absolute value is due to the fact that it is actually a complex value).

In contrast to the mechanical oscillation, here the resonance frequency is independent of the damping. As can be easily shown from relations (6) and (7), at the resonance point the phase displacement becomes zero in all components of the circuits.

In the case of the parallel-tuned RLC-circuit, we apply Kirchhoff's first law:

$$I_R + I_L + I_C = 0 \quad (10)$$

Because the function generator represents a constant voltage source (and not constant current), we differentiate equ. (10) with respect to time, use the identities (2) and we obtain

$$\ddot{U} + \frac{1}{RC}\dot{U} + \frac{1}{LC}U = 0 \quad (11)$$

With the ansatz $U(t) = U_0 \cdot \exp(i\omega t)$ and after discarding the imaginary part one directly obtains the resonance frequency $\omega_0 = \frac{1}{\sqrt{LC}}$. To determine the impedance for the parallel tuned circuit, one simply uses (10) with $I_R = I$ and $I(t) = \frac{U(t)}{Z}$ to obtain

$$\frac{U(t)}{Z_p} = \frac{U(t)}{X_L} + \frac{U(t)}{X_C} \Rightarrow \frac{1}{Z_p} = \left| \frac{1}{i\omega L} + i\omega C \right| \quad (12)$$

Applying Kirchhoff's first law on the complete circuit and regarding the LC-component as one element one gets

$$U_{\text{ext}} = U_R + U_{LC} \quad (13.1)$$

$$U_0 \cdot e^{i\omega t} = RI + Z_{LC}I \quad (13.2)$$

(here $Z_{LC} = Z_p$). Therefore the solution for the current is, after neglecting the imaginary part,

$$I(t) = I_0 \cos(\omega t + \phi) \quad (14)$$

with

$$I_0 = \frac{U_0}{\sqrt{R^2 + \left(\frac{\omega L}{1 - \omega/\omega_0} \right)^2}} \quad (15)$$

The phase displacement ϕ is given by

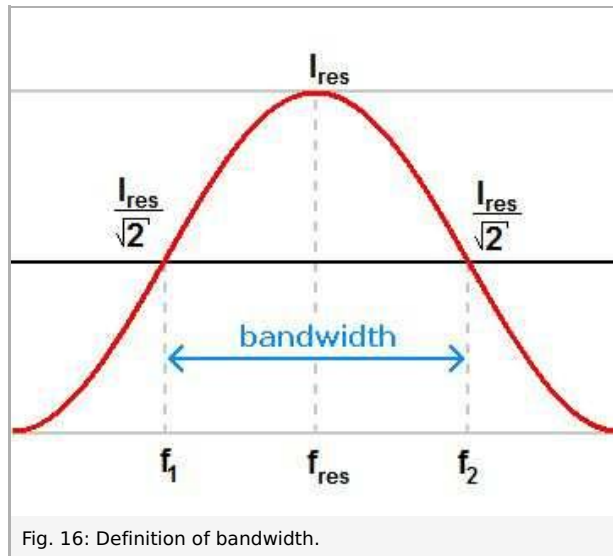
$$\tan(\phi) = \frac{1}{R \left(\frac{1}{\omega L} - \omega C \right)} \quad (16)$$

Comparing the calculations from above, the results are the following:
Both circuits (series- and parallel-tuned) have the same resonance frequency

$$f_{\text{res}} = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} \quad (17)$$

In the series-tuned case, the impedance tends to zero when the frequency is approaching the resonance frequency, which can be seen in the increase of current. In the parallel-tuned case, the impedance of the LC-component increases while approaching the resonance frequency, which can be seen in the decrease of current.

Another physical quantity, which describes the behaviour of a resonating system is the bandwidth B and the quality-factor Q . The bandwidth of a resonance curve is simply defined as the distance between the two points where the maximum amplitude $A_{\text{max}} = A_{\text{res}}$ at the resonance drops to a value $\frac{A_{\text{res}}}{\sqrt{2}}$ (see Fig. 3), so



$$B = f_2 - f_1 \tag{18}$$

The quality factor is given by Q

$$Q = \frac{f_{\text{res}}}{B} \tag{19}$$

In the series-tuned circuit, the quality factor can also be expressed as

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}} \tag{20}$$

which can be derived from the equations above (but usually one uses the relation $B = 2\delta$, where δ is the damping, which provides a much easier and faster way to obtain equ. (20)). One can see, that the resistor is responsible for the shape of the resonance curve, too.

In the parallel-tuned circuit, the quality factor, expressed through the parameters of the electrical components, is given by

$$Q = R \sqrt{\frac{C}{L}} \tag{21}$$

Results and evaluation

Calculate the resonance frequencies f_{res} for the combinations of components that you used during the experiment by using

$$f_{res} = \frac{1}{2\pi\sqrt{LC}}$$

and compare these to the values from your measurements.

Your data might look as follows:

	10 nF	47 nF	0.1 μF	0.47 μF
series circuit (measured)	10785 Hz	5025 Hz	3525 Hz	1550 Hz
parallel circuit (measured)	11000 Hz	5115 Hz	3585 Hz	1590 Hz
theoretical value	10273 Hz	4739 Hz	3249 Hz	1499 Hz

You will find that the measured values will be in good agreement with the prediction.

Now use the values from the parallel connection for a bandwidth and quality factor evaluation.

In the example data (here: $C = 0.47 \mu\text{F}$, $R = 470 \Omega$), a resonance frequency of 1588 Hz and a bandwidth of 605 Hz were determined, resulting in a quality factor of 2.63 (theoretical prediction: 2.1).

For the series circuit (here: $C = 0.47 \mu\text{F}$, $R = 47 \Omega$), a resonance frequency of 1550 Hz and a bandwidth of 428 Hz were determined, resulting in a quality factor of 3.62 (theoretical prediction: 4.8).

Your data may deviate from the prediction, as the values given for capacitance, inductance and resistance may not be exact. You can also perform a more precise calculation that accounts for the exact resistance value of the circuit, considering the different parts which contribute to the total resistance. These are the ohmic resistor R itself, the real part of the impedance at the resonance point, here simply denoted as R_{LC} , which is simply given by

$$R_{LC} = \frac{U_{res}}{I_{res}}$$

and the internal resistance of the function generator R_i , which is 2Ω . Therefore (here: series connection)

$$Q_{th} = \frac{1}{R_{tot}} \sqrt{\frac{L}{C}} \quad \text{with} \quad R_{tot} = R + R_i + R_{LC} = R + R_i + \frac{U_{res}}{I_{res}}$$

When using the 470Ω resistor, it is sufficient to use this value as an approximation, as the other contributions to the resistance are comparably small.

You may wish to repeat this evaluation for other combinations of components that you measured.

Application

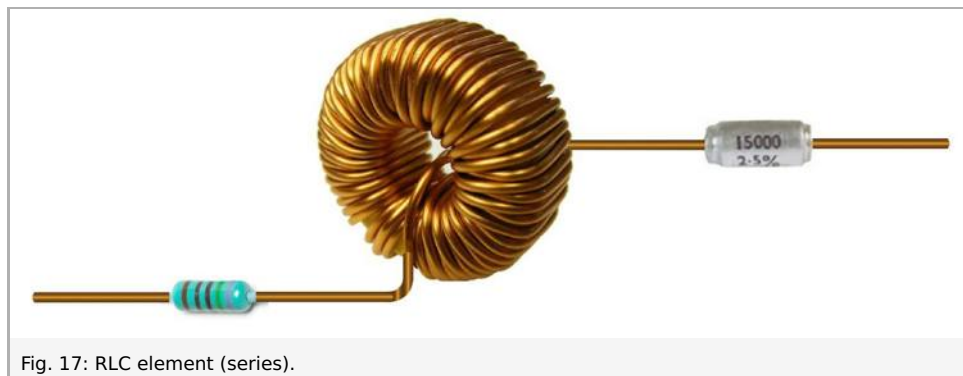


Fig. 17: RLC element (series).

RLC-circuits are used as frequency filters or resonators in electronic devices; e.g. in radio transmitters and receivers the frequency tuning is accomplished by setting the RLC-circuit to resonate on a special frequency.