Magnetic field of single coils / Biot-Savart's law

(Item No.: P2430201)

Curricular Relevance

Area of Expertise:	Education Level:	Topic:	Subtopic:	Experiment:
Physics	University	Electricity and Magnetism	Magnetic Field and Magnetic Forces	Magnetic field of single coils / Biot-Savart's law

Difficulty	Preparation Time	Execution Time	Recommended Group Size
Difficult	10 Minutes	20 Minutes	2 Students

Additional Requirements:

Experiment Variations:

Keywords:

Wire loop, Biot-Savart's law, Hall effect, magnetic field, induction, magnetic flux density

Introduction

Overview

The magnetic field along the axis of wire loops and coils of different dimensions is measured with a teslameter (Hall probe). The relationship between the maximum field strength and the dimensions is investigated and a comparison is made between the measured and the theoretical effects of position.

Fig 1: Experimental set-up for measuring a magnetic field.

Equipment

Position No.	Material	Order No.	Quantity
1	Induction coil,300 turns,dia.40mm	11006-01	1
2	Induction coil,300 turns,dia.32mm	11006-02	1
3	Induction coil,300 turns,dia.25mm	11006-03	1
4	Induction coil,200 turns,dia.40mm	11006-04	1
5	Induction coil,100 turns,dia.40mm	11006-05	1
6	Induction coil,150 turns,dia.25mm	11006-06	1
7	Induction coil, 75 turns,dia.25mm	11006-07	1
8	Conductors, circular, set	06404-00	1
9	Teslameter, digital	13610-93	1
10	Hall probe, axial	13610-01	1
11	PHYWE power supply, universal DC: 0...18 V, 0...5 A / AC: 2/4/6/8/10/12/15 V, 5 A	13500-93	1
12	Meter scale, l = 1000 mm	03001-00	1
13	Digital multimeter 2005	07129-00	1
14	Barrel base PHYWE	02006-55	2
15	Support rod PHYWE,square,l 250mm	02025-55	1
16	Distributor	06024-00	1
17	Right angle clamp expert	02054-00	1
18	Universal clamp	37715-00	1
19	G-clamp	02014-00	2
20	Lab jack, 200 x 200 mm	02074-01	1
21	Reducing plug 4mm/2mm socket, 2	11620-27	1
22	Connecting cord, 32 A, 500 mm, blue	07361-04	1
23	Connecting cord, 32 A, 500 mm, red	07361-01	2

Tasks

1. To measure the magnetic flux density in the middle of various wire loops with the Hall probe and to investigate its dependence on the radius and number of turns.
2. To determine the magnetic field constant μ_0
3. To measure the magnetic flux density along the axis of long coils and compare it with theoretical values.

Set-up and procedure

Set up the experiment as shown in Fig. 1. Operate the power supply as a constant current source, setting the voltage to 18 V and the current to the desired value. Measure the magnetic field strength of the coils (I = 1 A) along the z-axis with the Hall probe and plot the results on a graph. Make the measurements only at the centre of the circular conductors (I = 5 A). To eliminate interference fields and asymmetry in the experimental set-up, switch on the power and measure the relative change in the field. Reverse the current and measure the change again. The result is given by the average of the measured values.

Theory and evaluation

From Maxwell's equation

$$\int_K \vec{H} d\vec{s} = I + \int_F \vec{D} d\vec{f} \quad (1)$$

where K is a closed curve around area F, H is the magnetic field strength, I is the current flowing through area F, and D is the electric flux density, we obtain for direct currents ($D = 0$), the magnetic flux law:

$$\int_K \vec{H} d\vec{s} = I \quad (2)$$

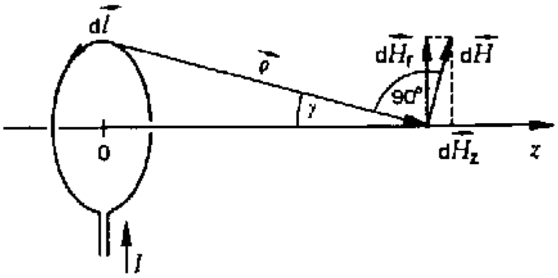

Fig. 2: Drawing for the calculation of the magnetic field along the axis of a wire loop.

With the notations from Fig. 2, the magnetic flux law (2) is written in the form of Biot-Savart's law:

$$d\vec{H} = \frac{I}{4\pi} \frac{d\vec{l} \times \rho}{\rho^3} \tag{3}$$

The vector $d\vec{l}$ is perpendicular to $\vec{\rho}$ in addition to this $\vec{\rho}$ and d$\vec{H}$ lie in the plane of the drawing, so that

$$dH = \frac{I}{4\pi\rho^2} dl = \frac{I}{4\pi} \frac{dl}{R^2 + z^2} \tag{4}$$

$d\vec{H}$ can be resolved into a radial dH_r and an axial dH_z component.

The dH_z components have the same direction for all conductor elements $d\vec{l}$ and the quantities are added; the dHr components cancel one another out, in pairs.

Therefore,

$$H_r(z) = 0 \tag{5}$$

and

$$H(z) = H_z(z) = \frac{I}{2} \frac{R^2}{(R^2 + z^2)^{3/2}} \tag{6}$$

along the axis of the wire loop, while the magnetic flux density

$$B(z) = \frac{\mu_0 \cdot I}{2} \cdot \frac{R^2}{(R^2 + z^2)^{3/2}} \tag{7}$$

where μ_0= 1.2566×10^{-6} H/m is the magnetic field constant. If there is a small number of identical loops close together, the magnetic flux density is obtained by multiplying by the number of turns n.

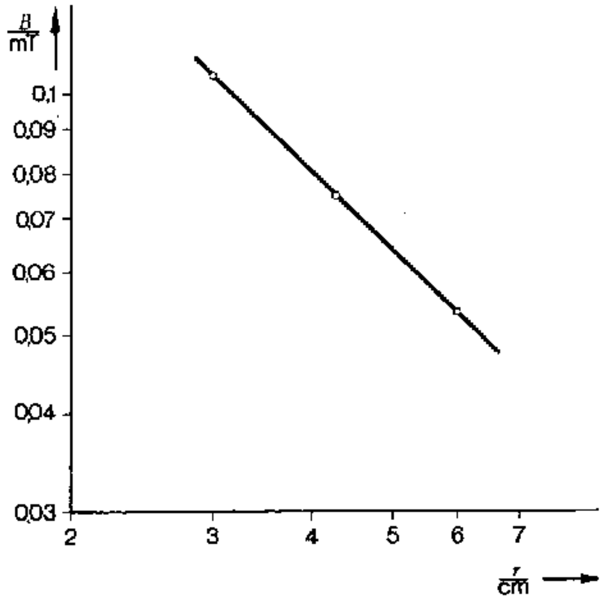

Fig. 4: Magnetic flux density at the centre of a single turn, as a function of the radius (current 5 A).

1. At the centre of the loop (z=0) we obtain

$$B(0) = \frac{\mu_0 \cdot n \cdot I}{2R} \quad (8)$$

Using the expression

$$B = A_1 \cdot n^{E_1}$$

and

$$B = A_2 \cdot R^{E_2}$$

the regression lines for the measured values in Fig. 3 and Fig. 4 give, for the number of turns, the following exponents E and standard errors:

$$E_1 = 0.96 \pm 0.04$$

and, for the radius (see equation (8))

$$E_2 = -0.97 \pm 0.02$$

2. Using the measured values from Fig. 3 and Fig. 4, and equation (8), we obtain the following average value for the magnetic field constant:

$$\mu_0 = (1.28 \pm 0.01) \times 10^{-6} \text{ H/m}$$

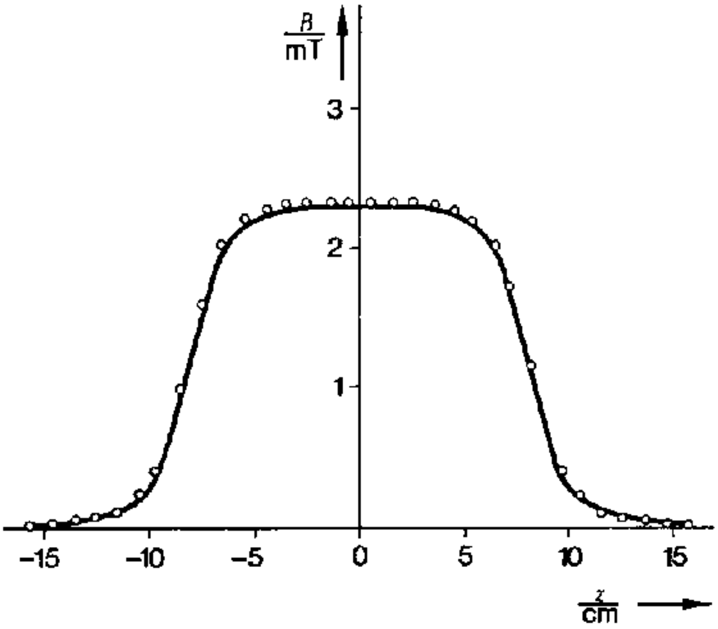

Fig. 5: Magnetic flux density along the axis of a coil of length l = 162 mm, radius R = 16 mm, and n = 300 turns; measured values (circles) and theoretical curve (continuous line) in accordance with equation (9).

3. To calculate the magnetic flux density of a uniformly wound coil of length l and n turns, we multiply the magnetic flux density of <u>one</u> loop by the density of turns *n/l* and integrate over the coil length.

$$B(z) = \frac{\mu_0 \cdot I \cdot n}{2l} \cdot \left(\frac{a}{\sqrt{R^2 + a^2}} - \frac{b}{\sqrt{R^2 + b^2}} \right) \quad (1)$$

where

$$a = z + l/2$$

and

$$b = z - l/2$$

The proportional relationship between magnetic flux density B and number of turns n at constant length and radius is shown in Fig. 6. The effect of the length of the coil at constant radius with the density of turns n/l also constant, is shown in Fig. 7.

Comparing the measured with the calculated values of the flux density at the centre of the coil,

$$B(0) = \frac{\mu_0 \cdot I \cdot n}{2l} \cdot \left(R^2 + \frac{l}{2}\right)^{-\frac{1}{2}}$$

gives the values shown in Table 1.

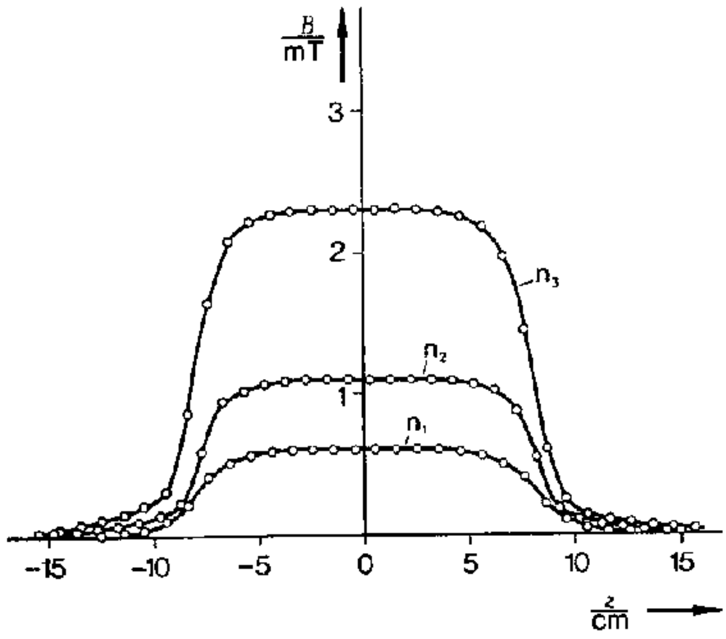

Fig. 6: Curve of magnetic flux density (measured values) along the axis of coil of length l = 160 mm, radius R = 13 mm and number of turns n_1 = 75, n_2 = 150 and n_3 = 300.

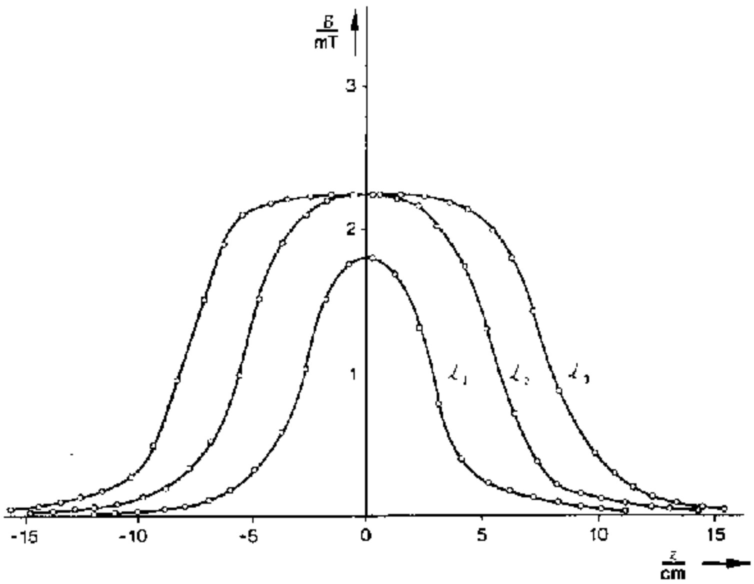

Fig. 7: Curve of magnetic flux density (measured values) for coils with a constant density of turns n/l, coils radius R = 20 mm, lengths l_1 = 53 mm, l_2 = 105 mm and l_3 = 160 mm.

Table 1: Comparison of the measured and the calculated values of the flux density.

n	l /mm	R /mm	B(0) /mT measured	B(0) /mT calculated
75	160	13	0.59	0.58
150	160	13	1.10	1.16
300	160	13	2.30	2.32
100	53	20	1.81	1.89
200	105	20	2.23	2.24
300	160	20	2.23	2.29
300	160	16	2.31	2.31