(Item No.: P2150100)

Curricular Relevance

Area of Expertise:	Education Level:	Topic:	Subtopic:	Experiment:
Physik	Klasse 7-10	Mechanik	Schwingungen und Wellen	

Difficulty	Preparation Time	Execution Time	Recommended Group Size
Difficult	10 Minutes	20 Minutes	2 Students

Additional Requirements:

Experiment Variations:

Keywords:

natural vibration, mass-spring system, harmonic sound intervals

Introduction

Overview

A tensioned metal string is made to vibrate. The vibrations of the string are optically scanned, the vibration process observed on the oscilloscope and the dependence of the frequency on the string tension and string length and the density of the material are investigated.

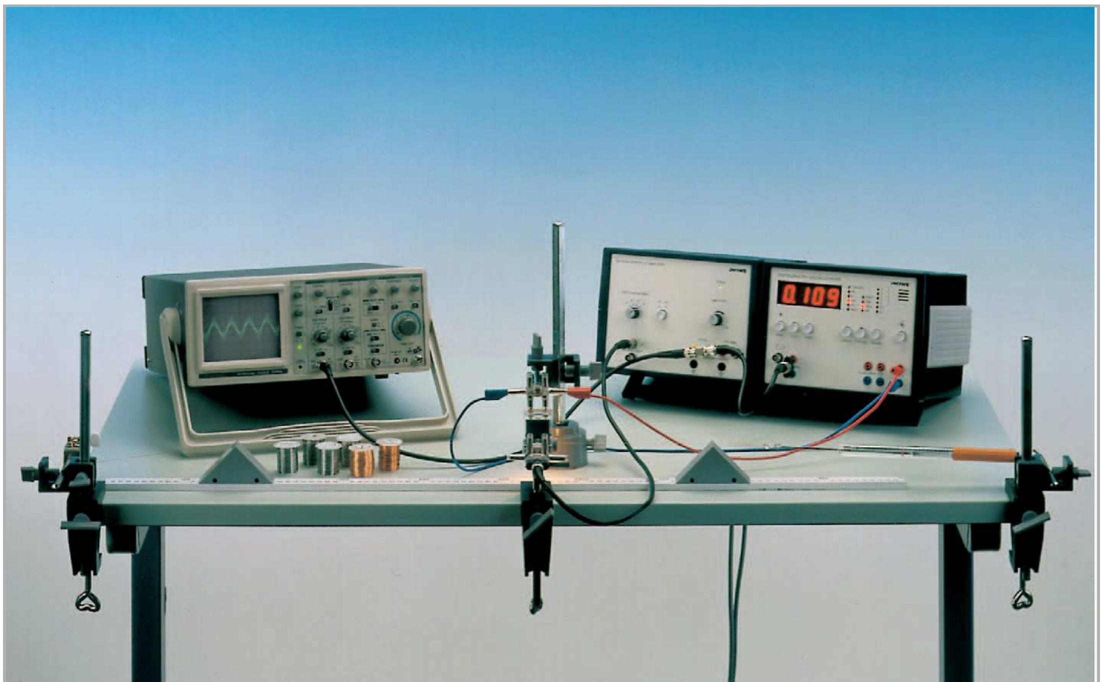

Fig. 1: Experimental set-up for measuring the frequency of vibration of strings.

Equipment

Position No.	Material	Order No.	Quantity
1	String tensioning device, w. stem	03431-01	1
2	Nickel wire, d = 0.3 mm, l = 100 m	06090-00	1
3	Kanthal wire, 19.1 Ohm/m, d = 0.3 mm, l = 100 m	06092-00	1
4	Constantan wire, 6.9 Ohm/m, d = 0.3 mm, l = 100 m	06101-00	1
5	Constantan wire, 4 Ohm/m, d = 0.4 mm, l = 50 m	06102-00	1
6	Copper wire, d = 0.4 mm, l = 50 m	06106-02	1
7	Copper wire, d = 0.5 mm, l = 50 m	06106-03	1
8	Barrel base PHYWE	02006-55	1
9	Bench clamp PHYWE	02010-00	3
10	Support rod, stainless steel, l = 250 mm, d = 10 mm	02031-00	3
11	Right angle clamp expert	02054-00	3
12	Rod with hook	02051-00	1
13	Sign holder	02066-00	2
14	Fish line, l. 100m	02090-00	1
15	Meter scale, l = 1000 mm	03001-00	1
16	Spring balance 100 N	03060-04	1
17	Striking hammer	03429-00	1
18	Photoelement	08734-00	1
19	Lampholder E10, case G1	17049-00	1
20	Filament lamp 6 V/3 W, E10, 10 pcs.	35673-03	1
21	Distributor	06024-00	1
22	30 MHz digital storage oscilloscope with colour display,2 x BNC cables l = 75 cm incl.	11462-99	1
23	LF amplifier, 220 V	13625-93	1
24	Universal Counter	13601-99	1
25	Plug with push-on sleeve	07542-04	1
26	Adaptor, BNC socket/4 mm plug	07542-20	1
27	Adapter, BNC-plug/socket 4 mm	07542-26	2
28	Connector, T type, BNC	07542-21	1
29	Adapter, BNC socket/4 mm plug pair	07542-27	1
30	Screened cable, BNC, l = 750 mm	07542-11	1
31	Screened cable, BNC, l 250 mm	07542-10	1
32	Connecting cord, 32 A, 750 mm, red	07362-01	1
33	Connecting cord, 32 A, 750 mm, blue	07362-04	1

Tasks

1. Measure the frequency of a string (e.g. constantan, ⌀ = 0.4 mm) as a function of the tensioning force and the length of the string.
2. Measure the frequency for various types and cross-sections of string, at a fixed tension and string length.

Set-up and procedure

The string is laid across 2 triangular sliders and clamped between a fixed hook and a spring balance, as shown in Fig. 1. The spring balance is attached to a string tensioner with fishing line. The tensioning force should be no greater than 30 N–40 N (depending on the material from which the string is made) as otherwise the string may break.

The string length can be set by moving the triangular sliders along the measuring scale. If a piece of the string outside these sliders vibrates as well it can be stopped by gently laying a finger on it (this must not, however, alter the string tension).

The vibration of the string is optically scanned midway between the sliders.

The signal from the photo-cell, fitted with an aperture slit, is amplified and transmitted to the oscilloscope and the counter.

A gentle tap with the rubber hammer is sufficient to start the string vibrating. The counter is only started once the harmonics have died away.

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Theory and evaluation

First of all, we will deal with the propagation of a transverse wave over a tensioned string (Fig. 2).

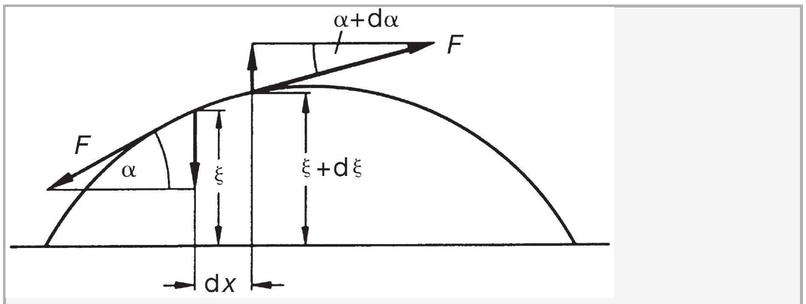

Fig. 2: Deriving the differential equation of transverse waves on a tensioned string.

Using the notations in Fig. 2:

When the string component $\mathrm{d}x$ is deflected from the rest position by ξ, the restoring force F_ξ is obtained from:

$$F_\xi = F\sin(\alpha) - F\sin(\alpha + \mathrm{d}\alpha) \tag{1}$$

α is the angle between the x axis and the ξ axis, so that:

$$\tan\alpha = \frac{\partial \xi}{\partial x} \tag{2}$$

The deflection ξ may be so small that

$$\alpha = \sin\alpha = \tan\alpha \tag{3}$$

From (2), therefore, we obtain:

$$\mathrm{d}\alpha = \frac{\partial^2 \xi}{\partial x^2} \tag{4}$$

and from (1)

$$F_\xi = -F\frac{\partial^2 \xi}{\partial x^2}\mathrm{d}x \tag{5}$$

The mass of the string element $\mathrm{d}x$ is

$$\mathrm{d}m = \rho q\, \mathrm{d}x \tag{6}$$

where q = cross section, ρ = the density of the material from which the string is made.

From the equation of motion

$$\mathrm{d}m \cdot \frac{\partial^2 \xi}{\partial t^2} = -F_\xi \tag{7}$$

we obtain, with (5) and (6), the wave equation

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{F}{q \cdot \rho} \cdot \frac{\partial^2 \xi}{\partial x^2} \tag{8}$$

$$c = \sqrt{\frac{F}{q \cdot \rho}} \tag{9}$$

is the propagation velocity of a transverse wave over the tensioned string.

The fundamental frequency of a string of length l tensioned at both ends is therefore

$$f = \frac{1}{2l}\sqrt{\frac{F}{q \cdot \rho}} \tag{10}$$

1. The relationship between fundamental frequency and the string length and tensioning force is measured on a constantan wire of 0.4 mm diameter.

   The regression line for the measured values in Fig. 3, using

   $$f = A \cdot l^B$$

   gives the exponent

   $$B = -1.01 \pm 0.03 \qquad \text{(see(10))}$$

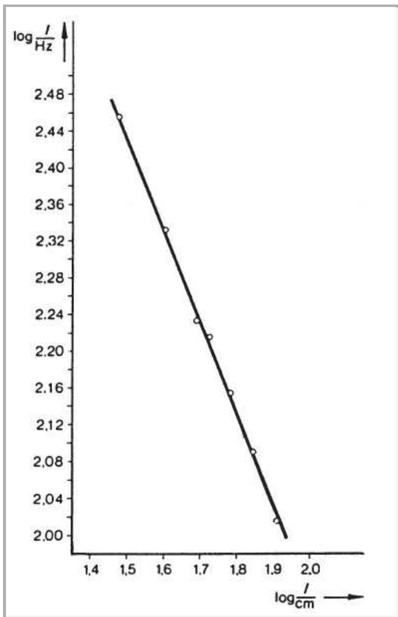

Fig. 3: Fundamental frequency f as a function of string length l at a given tensioning force F = 30 N.

This relationship is familiar from music; the measurements were:

Musical interval	l / cm	f / Hz
fundamental 1/1	80	104
fourth 3/4	60	143
fifth 2/3	53.3	165
octave 1/2	40	215

The regression line for the measured values in Fig. 4, using

$$f = A \cdot F^B$$

gives the exponent (cf. equation (10))

$$B = 0.48 \pm 0.02$$

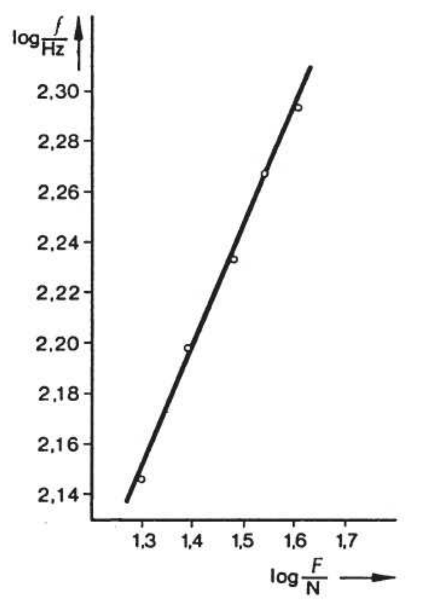

Fig. 4: Fundamental frequency f as a function of tensioning force F at a fixed string length l of 50 cm.

2. The relationship between the fundamental frequency and the density of the material is measured with nickel (ρ = 8.9 g/cm³), constantan (ρ = 8.8 g/cm³) and kanthal (ρ = 7.1 g/cm³) wires, each 0.3 mm diameter.

   The regression line for the measured values in Fig. 5, using

   $$f = A \cdot \rho^B$$

   gives the exponent (cf. equation (10))

   $$B = -0.51 \pm 0.01.$$

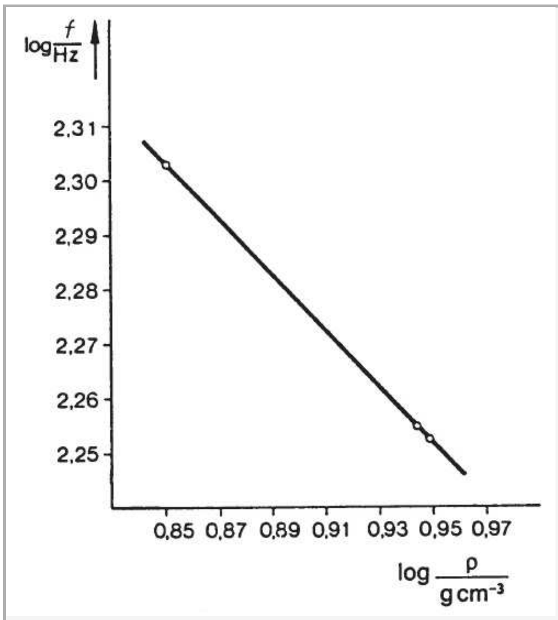

Fig. 5: Relationship between the fundamental frequency f and the density ρ of the material for a diameter of 0.3 mm, a force of 20 N and a length of 50 cm.

To determine the relationship between the fundamental frequency and the wire cross-section $q = \pi r^2$ two copper wires (ρ = 8.9 g/cm3) 0.4 mm and 0.5 mm diameter are available. The nickel wire has the same density and a diameter of 0.3 mm.

The regression line for the measured values in Fig. 6, using

$$f = A \cdot r^B$$

gives the exponent (cf. equation (10))

$$B = -101 \pm 0.02.$$

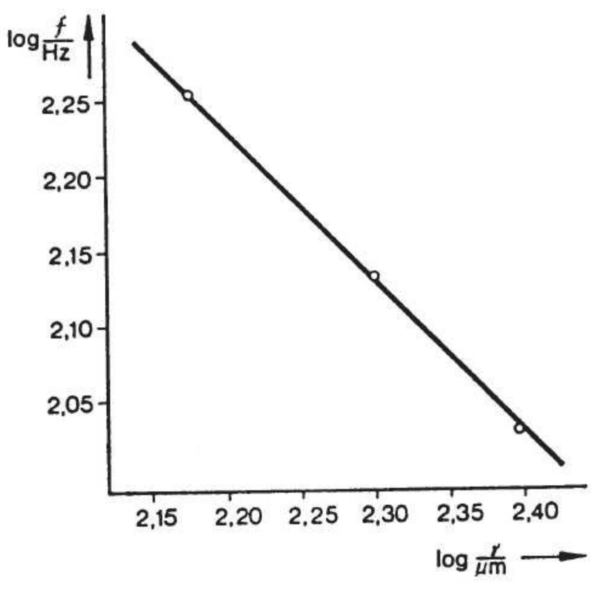

Fig. 6: Relationship between the fundamental frequency f and the wire radius r for a length of 50 cm and a force of 20 N.