Newton's second law with the demonstration track and "measure Dynamics" (Item No.: P2130380)

Curricular Relevance

| Area of Expertise: ILIAS | Education Level: Physik | Topic: Hochschule | Subtopic: Mechanik | Experiment: Eindimensionale Bewegung |

Difficulty

Intermediate

Preparation Time

20 Minutes

Execution Time

10 Minutes

Recommended Group Size

2 Students

| Additional Requirements: | Experiment Variations: |

Keywords:

velocity, acceleration, force, gravitational acceleration, kinetic energy, potential energy

Introduction

Overview

A mass, which is connected to a cart via a silk thread, drops to the floor. The resulting motion of the cart will be recorded by way of a video camera and evaluated with the "measure Dynamics" software. The re-lationship between distance and time, velocity and time, and the relationship between mass, accelera-tion, and force will be determined for a uniformly accelerated rectilinear motion with the aid of the dem-onstration track. In addition, the conversion of potential energy into kinetic energy will be represented graphically as well as by integrating the various forms of energy into the video in the form of bars.

Figure 1: Experiment set-up

Equipment

Position No.	Material	Order No.	Quantity
1	Starter system for demonstration track	11309-00	1
2	Magnet w.plug f.starter system	11202-14	1
3	Tube with plug	11202-05	1
4	Plasticine, 10 sticks	03935-03	1
5	Silk thread, l = 200 m	02412-00	1
6	Weight holder for slotted weights	02204-00	1
7	Needle with plug	11202-06	1
8	Demonstration track, aluminium, 1.5 m	11305-00	1
9	Cart, low friction sapphire bearings	11306-00	1
10	Holder for pulley	11305-11	1
11	End holder for demonstration track	11305-12	1
12	Portable Balance, OHAUS CS2000	48917-93	1
13	Slotted weight, black, 10 g	02205-01	4
14	Circular labels, red, pkg. of 416	06305-04	1
15	Software "Measure Dynamics", single user	14440-61	1
16	Pulley for demonstration track	11305-10	1

Tasks

1. Task 1: Determination of the distance covered as a function of time.

2. Task 2: Determination of the velocity as a function of time.

3. Task 3: Integration of the velocity into the video.

4. Task 4: Graphical representation of the conversion of potential energy into kinetic energy while demon-strating the law of conservation of energy.

5. Task 5: Integration of the potential, kinetic, and total energy into the video.

Set-up and procedure

The experiment set-up is shown in Fig. 1. Fasten the starter system to the track so that the cart does not get any initial momentum. In addition, it must be ensured that the falling mass m_1, which must be connected to the cart of the mass m_2 by way of a silk thread, is led over a pulley. It must also be ensured that the mass m_1 does not swing before or during the measurement. It must drop to the floor freely, i.e. without touching a corner of the table or similar. The mass of the cart m_2 can be varied by fastening slotted weights to the rod of the cart. This ensures a symmetrical distribution of the weight, which is essential for optimum gliding. In addition, a big, circular, coloured disc is attached to the rod of the cart for the subsequent video analysis. The falling mass m_1, which acts on the cart as an accelerating force, can be varied by changing the number of weights on the weight holder.

Prior to starting the experiment, the mass of the cart without any weights must be determined and the end holder must be positioned at the end of the track. It is absolutely essential to check the horizontal alignment of the track prior to starting the measurement. If necessary, adjust it with the aid of the three adjusting screws on which the track rests.

In terms of the video that will be recorded, the following must be taken into consideration concerning the setting and positioning of the camera:

- Set the number of frames per second to approximately 30 fps.

- Select a light-coloured, homogeneous background.

- Provide additional lighting for the experiment.

- The experiment set-up should be in the centre of the video. To ensure this, position the video camera on a tripod centrally in front of the experiment set-up.

- The experiment set-up should fill the video image as completely as possible.

- The optical axis of the camera must be parallel to the experiment set-up (no movement in the y-direction).

- For scaling, the drop distance of the mass m_1 must be determined and marked on the track in a clearly visible manner.

Then, the video recording process and the experiment can be started.

Theory and evaluation

Theory

Newton's law of motion for a mass point of the mass m that is subject to a force F is given by the follow-ing relationship:

$$\vec{F} = m \cdot \vec{a}$$

with the acceleration

$$\vec{a} = \frac{d^2\vec{r}}{dt^2}$$

and the position $\vec{r}$. The velocity $\vec{v}$, which is obtained by the application of a constant force $\vec{F}$, is given as a function of time by way of the expression

$$\vec{v}(t) = \frac{\vec{F}}{m} \cdot t$$

with the initial condition

$$\vec{v}(t=0) = 0$$

Assuming that

$$\vec{v}(t=0) = 0\,; \vec{r}(t=0) = 0,$$

the position $\vec{r}$ of the force $\vec{F}$ that acts on a mass point m is

$$\vec{r}(t) = \frac{1}{2} \cdot \frac{\vec{F}}{m} \cdot t \tag{0}$$

In the present case, the motion is unidimensional (linear motion) and the force F, which is applied due to the mass m_1, is

$$|\vec{F}| = m_1 \cdot |\vec{g}| = m_1 \cdot g,$$

where g is the gravitational acceleration. With the total mass that is to be moved $m = m_1 + m_2$ (= falling mass m_1 + total mass m_2 of the cart) the equation of motion is given by

$$(m_1 + m_2) \cdot |\vec{a}| = m_1 \cdot g \tag{1}$$

The velocity is

$$|\vec{v}(t)| = v = \frac{m_1 \cdot g}{m_1 + m_2} \cdot t \tag{2}$$

and the position is

$$|\vec{r}(t)| = s(t) = \frac{1}{2} \cdot \frac{m_1 \cdot g}{m_1 + m_2} \cdot t^2 \tag{3}$$

Evaluation and results

The evaluation process is explained based on the following model experiment. The accelerating distance is 0.76 m, the accelerating mass m_1 is 10 g, the total mass m_2 of the cart is 395 g (including the needle with plug, holding magnet with plug, needle, and tab).
Transfer the video that has been recorded to the computer. Then, start "measure Dynamics" and open the video under "File" – "Open video ...". Mark the start of the experiment ("Start selection" and "Time zero") and the end of the experiment ("End selection") in the video for further analysis via the menu line above the video. The experiment begins with the start of the cart and it ends when the cart reaches the mark at which the accelerating mass touches the floor. Then, mark the accelerating distance, which has been measured and marked beforehand, with the scale that appears in the video by way of "Video analysis" – "Scaling ..." – "Calibration" and enter the resulting length into the input window. In addition, enter the frame rate that has been set for the experiment (in this case 30) under "Change frame rate". Position the origin of the system of coordinates on the start point of the cart under "Origin and direction" and turn it by right-clicking so that the cart moves in the positive x direction. Then, the actual motion analysis can be started under "Video analysis" – "Automatic analysis" or "Man-ual analysis". For the

automatic analysis, we recommend selecting "Motion and colour analysis" on the "Analysis" tab. Under "Options", the automatic analysis can be optimised, if necessary, e.g. by changing the sensitivity or by limiting the detection radius. Then, look for a film position in the video where the cart is perfectly visible. Click the coloured disc of the cart. If the system recognises the object, a green rec-tangle appears and the analysis can be started by clicking "Start". If the automatic analysis does not lead to any satisfying results, the series of measurements can be corrected under "Manual analysis" by mark-ing the coloured disc manually. If the accelerating mass m1 is so small that the cart moves only very slightly in between two individual frames, the sampling rate (step) of the experiment should be changed. If the sampling rate (step) is set to "5", for example, only every fifth frame will be used for the evaluation.

Task 1: Determination of the distance covered as a function of time.

In order to display the curve of the distance covered as a function of time, select "Display" and "Dia-gram", click "Options", delete all of the already existing graphs, and select the graphs t (horizontal axis) – x (vertical axis). This leads to:
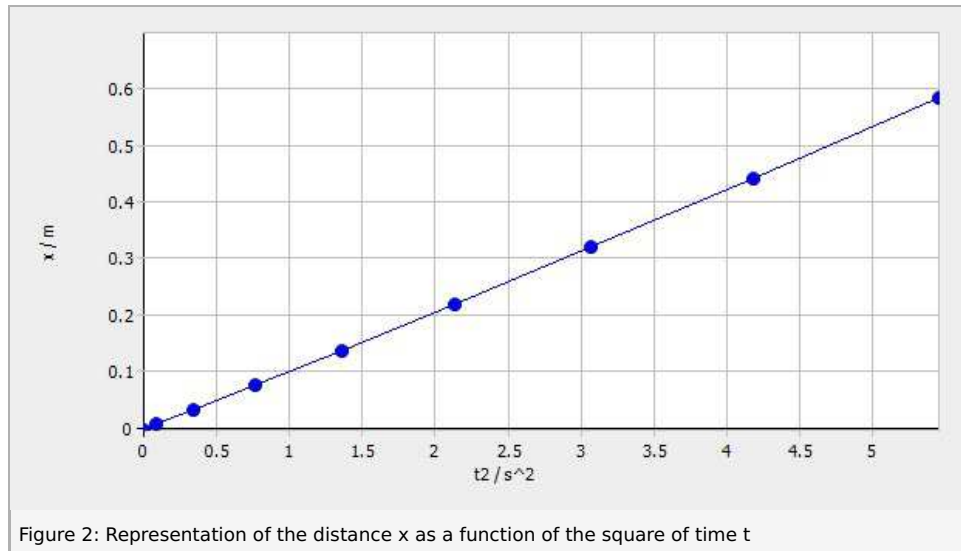

Figure 2: Representation of the distance x as a function of the square of time t

As it could have been expected from equation (0), Figure 3 shows a quadratic relationship. In order to examine it in greater detail, it is advisable to consider the dependence of the distance covered on the square of the time. In order to be able to visualise this in graphical form, the worksheet must be ex-tended by clicking "New column" in the table menu line. Then, enter "t2" (unit: "s^2"; formula: "t^2") into the new column. As a result, the t2-x diagram can be displayed in the same manner as the t-x diagram. The following results:
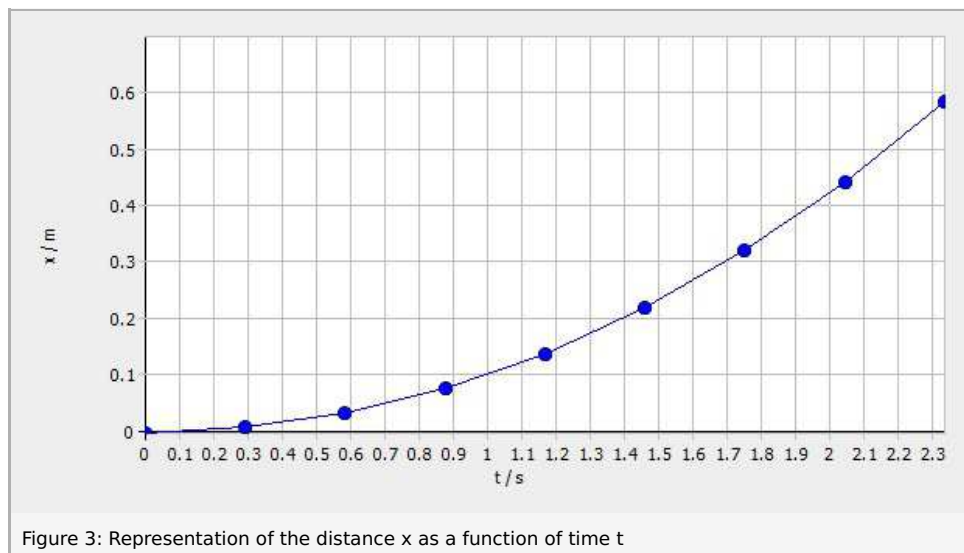

Figure 3: Representation of the distance x as a function of time t

Figure 2 shows that the distance increases linearly with the square of the time. Clicking "Options" in the menu line of the diagram and selecting the tab "Linear regression" will add a regression line to the dia-gram and the corresponding function will be displayed in the menu window. In this case here, the gradient of the curve is 0.107. Equation (0) leads to:

$$F = 2 \cdot (m_1 + m_2) \cdot 0.107 = 0.087 \, \text{N}.$$

This corresponds approximately to the weight of the accelerating mass m_1:

$$F = m_1 \cdot g = 0.098 \, \text{N}$$

The linear relationship between the distance covered and the time, which can be expected based on the theory, can be confirmed with this experiment.

Task 2: Determination of the velocity as a function of time.

In the same manner, it is also possible to visualise the velocity in the x direction as a function of time.
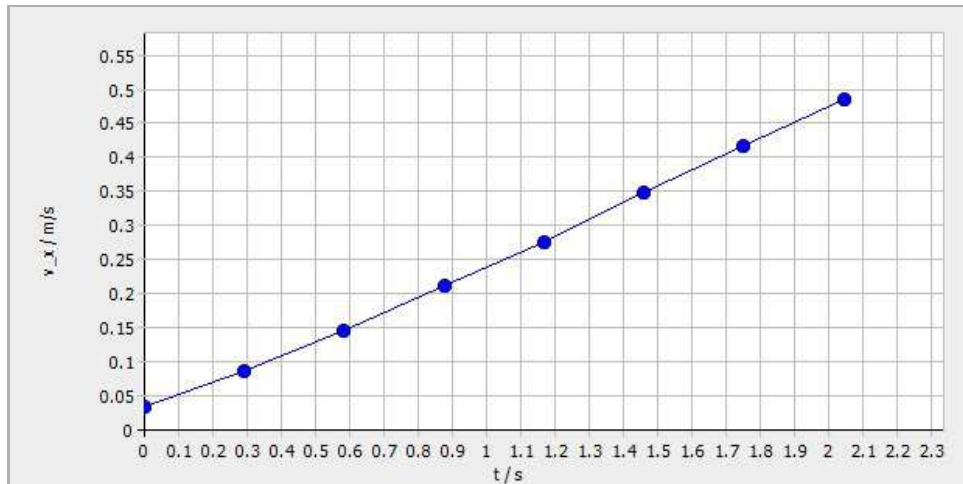This leads to:

Figure 4: Acceleration in the direction of movement (x-direction) as a function of time t

Figure 4 shows that the velocity increases linearly over time. The resulting regression line has a gradient of 0.2234. Based on equations (1) and (2), this corresponds to the acceleration a. Based on the theory, an acceleration of

$$a = \frac{m_1 \cdot g}{m_1 + m_2} = 0.24 \text{ m/s}^2$$

could be expected. This means that our experimental value corresponds approximately to the value that could have been theoretically expected.

Task 3: Integration of the velocity into the video

The video becomes even clearer when the velocity is integrated. To do so, open "Filters and labels ..." under "Display", click "Add new filter", and select the "Velocity arrow". Under "Filter configuration", select the tab "Limitations" and "Filter visible". Then, select "0" as the "Start selection" and "-1" as the "End selection" under "Cutting (timeline)". On the tab "Icon", select "Arrow" as the icon, since the velocity is a vector. Select "0" under "Trace length". This means that the icon will remain visible in the entire video. Adjust the "Step" so that approximately five to eight velocity vectors will be added to the video. More vectors would clutter the video. On the "Data source" tab, select the table of the cart under "Starting point" and "0" as the "Time increment". Select "x" as the "x-coordinate". Then, select "Fixed value" as the "ycoordinate" and position it so that the vectors will be displayed just above the experiment. Next, choose a suitable "Stretch factor" to ensure that the vectors are neither too short nor overlapping. Select the table of the cart once again under "End point" and set the "Time increment" to "0". Select "v_x" as the xcoordinate and "Fixed value" as the y-coordinate. Set it to "0". Then, activate "User-defined scale". The arrows can be labelled under "Display" – "Paint ..." – "Text". "Export" – "Picture series ..." then leads to:
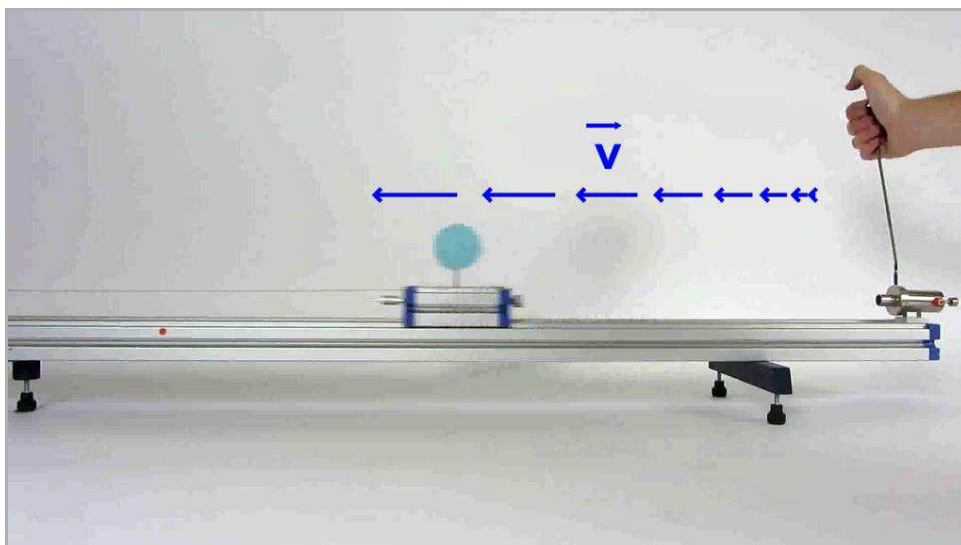

Figure 5: Integration of the velocity vectors into the video

Figure 5 shows the following: The direction of the arrow indicates the direction of the velocity and, thereby, the direction of movement of the cart. The length of the arrows increases approximately in a linear manner with the length of the arrow being a measure of the absolute value of the velocity. The linear increase in the length of the arrows underlines the result of task 2.

Task 4: Graphical representation of the conversion of potential energy into kinetic energy while demonstrating the law of conservation of energy.

The measurements that have been performed so far can now be used to visualise the conversion of potential energy into kinetic energy. For this purpose, the worksheet needs three new columns. Enter the kinetic energy (name: "E_kin"; unit: "J"; formula: "0.5*(m_1+m_2)*(v_x)^2" into the first column and the potential energy (name: "E_pot"; unit: "J"; formula: "m_1*g*(h-x)") with the length h from the start of the cart until the end of the accelerated distance into the second column. Enter the total energy (name: "E_total"; unit: "J"; formula: "E_kin+E_pot") into the third column.
Then, the three graphs t-E_kin, t-E_pot, and t-E_total are displayed together in one diagram. This leads to:
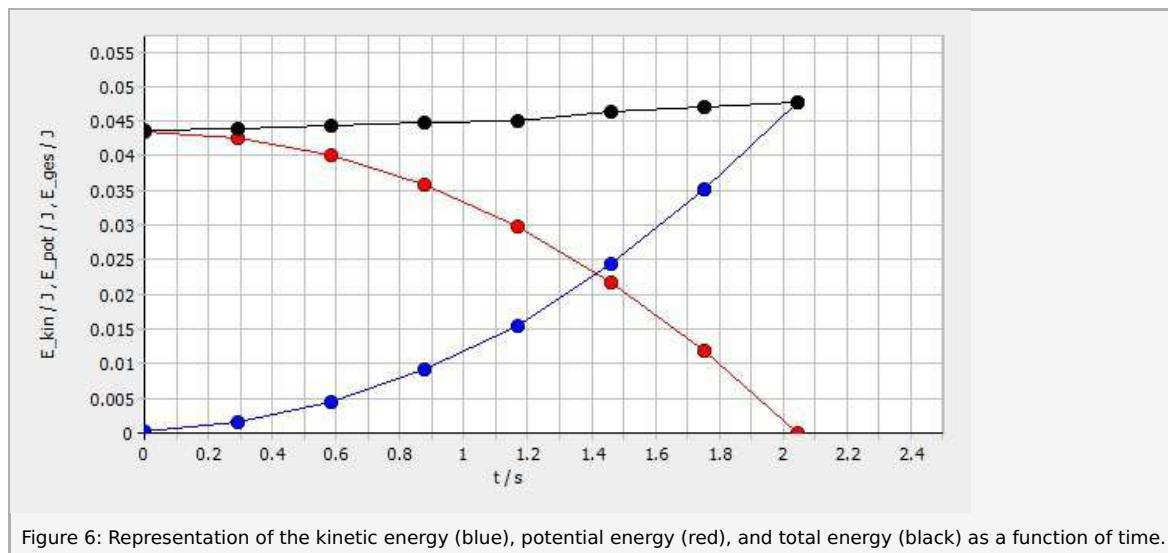

Figure 6: Representation of the kinetic energy (blue), potential energy (red), and total energy (black) as a function of time.

Figure 6 shows that, at the beginning of the experiment, only the potential energy of the mass m_1 exists. After the start of the experiment, the mass m_1 drops to the floor, thereby accelerating the cart of the mass m_2 and the accelerating mass m_1. As a result, the potential energy is converted into kinetic energy. This continues until the mass m_1 has no potential energy left. The total energy, i.e. the sum of potential and kinetic energy, remains approximately constant.

Task 5: Integration of the potential, kinetic, and total energy into the video.

In a similar manner, the conversion of potential energy into kinetic energy can also be integrated into the video. Once again, select the "Velocity arrow" under "Display" – "Filters and labels ...". In the "Filter con-figuration" dialogue box, activate "Filter visible" on the "Limitations" tab and specify "Cutting (timeline)" for the filter. Then, select the "Line" icon under "Change icon" on the "Icon" tab. We recommend select-ing a sufficiently large width of the line under "Options" under "Change icon". Set the "Trace length" and "Step" to "1". On the "Data source" tab, select the table of the cart under "Starting point" and "0" as the "Time increment". Select "Fixed value" as the "x- and y-coordinate" and position this value so that the bar is clearly visible. Select the table of the cart once again under "End point" and set the "Time incre-ment" to "0". Once again, select "Filed value" as the "x-coordinate" and set it to "0". Select "E_total" as the "y-coordinate" and activate "User-defined scale". Then, define a suitable "Stretch factor".

Add a second bar for the total energy in the same manner next to the first one. Add a bar for the kinetic energy of the cart in the same manner and with the same coordinates over the bar of the total energy (select "E_kin" as the "y-coordinate" under "End point"). The overlapping bars of the total energy and kinetic energy indirectly provide the bar for the potential energy, which corresponds to the subtraction of the kinetic energy from the total energy. The bars can be labelled under "Display" – "Paint ..." – "Text". "Export" – "Picture series ..." then leads to:
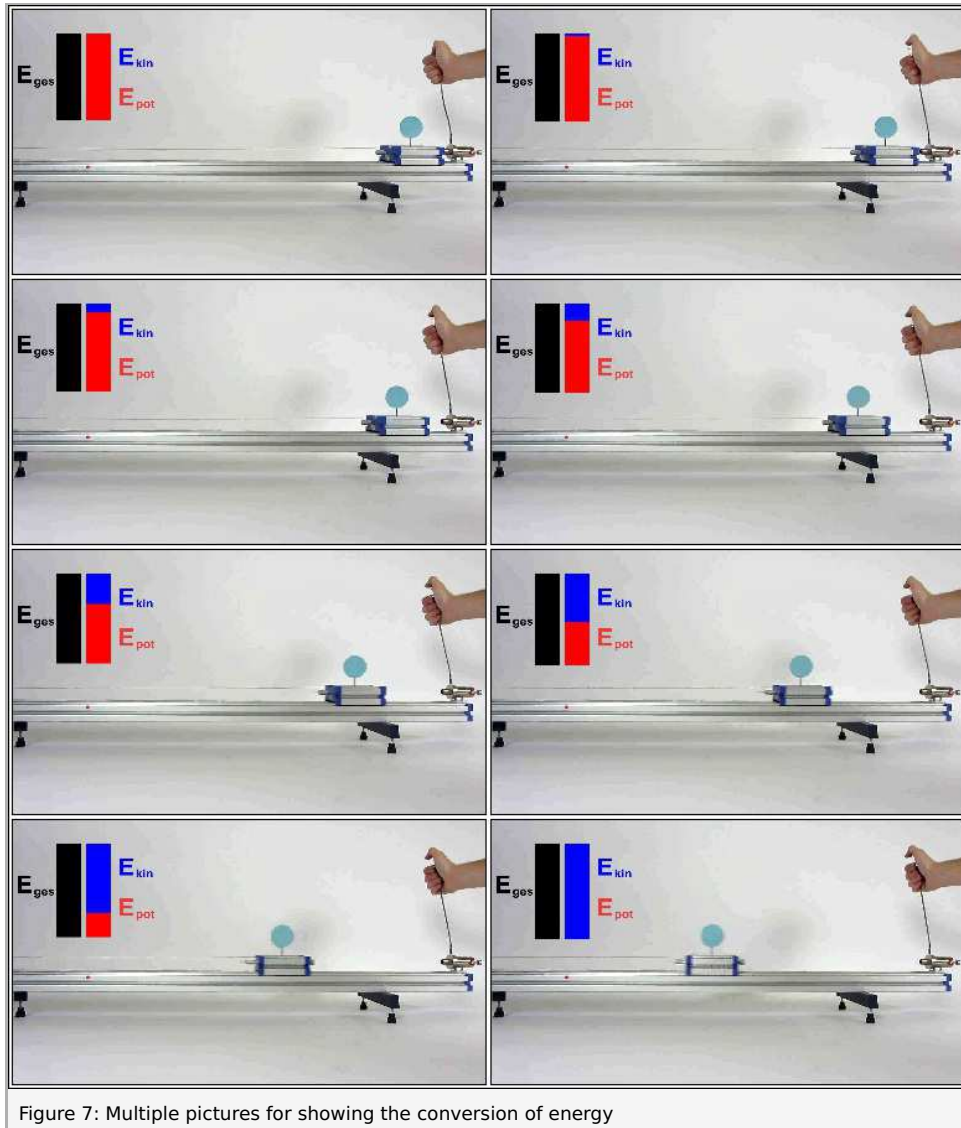
Figure 7: Multiple pictures for showing the conversion of energy

Figure 7 shows that the potential energy is completely converted into kinetic energy while the total energy remains constant. This is a clear confirmation of the conversion of potential energy into kinetic en-ergy and at the same time also a demonstration of the law of conservation of energy.