Conservation of momentum in multiple central inelastic collisions with the demonstration track and timer 4 – 4 (Item No.: P1199905)

Curricular Relevance

Area of Expertise:	Education Level:	Topic:	Subtopic:	Experiment:
Physik	Klasse 7-10	Mechanik	Lineare Bewegung	

Difficulty	Preparation Time	Execution Time	Recommended Group Size
Intermediate	10 Minutes	20 Minutes	2 Students

Additional Requirements:

Experiment Variations:

Keywords:

Inelastic collision, multiple collisions, conservation of momentum, conservation of energy, kinetic energy, deformation energy

Overview

Introduction

During inelastic collisions, kinetic energy is withdrawn from the system and converted into internal energy, which is no longer available for motion. As a result, the kinetic energy decreases with every collision. During this process, the objects usually undergo deformation before they continue to move in a joint manner with a momentum that corresponds to the sum of the individual momenta before the collisions.

Educational objective

When two carts collide in an inelastic manner, they stick together and continue to move in one direction at the same velocity. The momentum of the motion corresponds to the sum of the individual momenta before the collision. There may also be several collisions, but the momentum is still conserved:

$$p_1 + p_2 + p_3 + \ldots \ = \ p' + p_3 + \ldots \ = \ p'' + \ldots$$

The kinetic energy is reduced after every inelastic collision, which is due to the deformation of the plasticine between the two carts. However, the total energy of the system is conserved if the deformation energies $\Delta E^{(i)}$ are taken into consideration:

$$E_{\text{kin}} = E'_{\text{kin}} + \Delta E' = E''_{\text{kin}} + \Delta E_1 + \Delta E'' = \ldots .$$

Related topics

Experiment P1199805 "Conservation of momentum in multiple central elastic collisions" can be performed for comparison. In addition, inelastic collisions between two carts in motion towards one another were demonstrated in experiment P1199705 "Conservation of
momentum in a central inelastic collision".

Figure 1: Experiment set-up

Equipment

Position No.	Material	Order No.	Quantity
1	Timer 4-4	13604-99	1
2	Starter system for demonstration track	11309-00	1
3	Demonstration track, aluminium, 1.5 m	11305-00	1
4	Cart, low friction sapphire bearings	11306-00	3
5	Light barrier, compact	11207-20	3
6	Portable Balance, OHAUS CS2000E	48911-00	1
7	End holder for demonstration track	11305-12	1
8	Weight for low friction cart, 400 g	11306-10	3
9	Magnet w.plug f.starter system	11202-14	1
10	Shutter plate for low friction cart, width: 100 mm	11308-00	3
11	Needle with plug	11202-06	3
12	Tube with plug	11202-05	3
13	Slotted weight, black, 10 g	02205-01	4
14	Slotted weight, black, 50 g	02206-01	3
15	Holder for light barrier	11307-00	3
16	Connecting cord, 32 A, 1000 mm, red	07363-01	3
17	Connecting cord, 32 A, 1000 mm, yellow	07363-02	3
18	Connecting cord, 32 A, 1000 mm, blue	07363-04	3
19	Slotted weight, silver bronze, 10 g	02205-02	4
20	Slotted weight, silver bronze, 50 g	02206-02	3
21	Plasticine, 10 sticks	03935-03	1
22	Slotted weight, blank, 1 g	03916-00	

Tasks

1. Determination of the momenta before and after two inelastic collisions between a moving cart and two carts at rest.
2. Determination of the kinetic energy before and after two inelastic collisions between a moving cart and two carts at rest.

Set-up and procedure

Set-up

Set the experiment up as shown in Figure 1:

1. In order to compensate for slight friction effects, the track must be slightly inclined by way of the adjusting screws at the track bases so that one cart is still just about prevented from rolling to the right. For adjustment purposes, it is also possible to let one cart roll along the track with an initial momentum and to compare the light barrier shading times.
2. Position the starter system at the left end of the track. Please note that, in order to start the cart with an initial momentum, the starter system must be installed so that the cart receives an impulse from the ram of the starter system (Fig. 2).

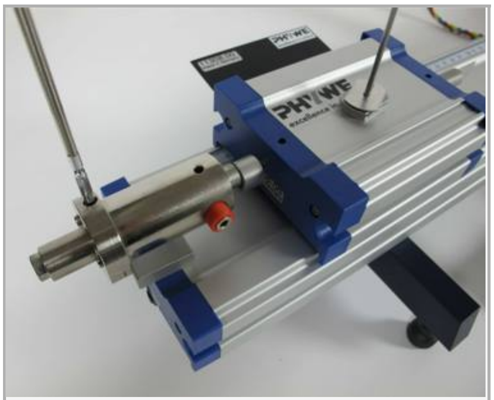

Fig. 2: Starter system for providing the necessary impulse

3. Attach a plasticine-filled tube to the end holder at the right-hand end of the track in order to stop the cart without a strong impact (Fig. 3).

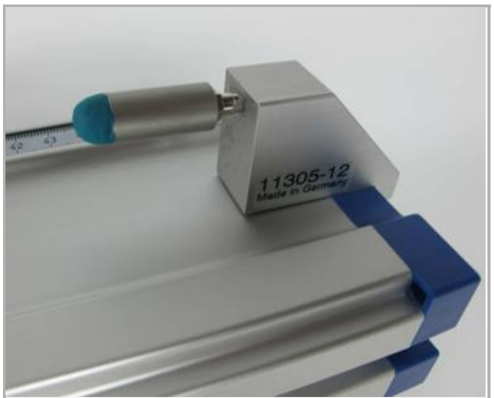

Fig. 3: End holder with plasticine

4. Fasten the three light barriers in the light barrier holders and position them at the 30 cm mark, 70 cm mark, and 110 cm mark on the track. The light barrier that is closest to the starter system is light barrier 1, the one in the middle is light barrier 2, and the one closest to the end holder is light barrier 3.
5. Connect the light barriers from the left to the right to the sockets in the fields "1" to "3" of the timer. In doing so, connect the yellow sockets of the light barriers to the yellow sockets of the measuring instrument, the red sockets to their red counterparts, and the blue sockets of the light barriers to the white sockets of the timer (see Fig. 4).

Fig. 4: Connection of the light barriers

6. In order to select the triggering edge, push the slide switch above field "1" of the timer to the right, i.e. to "falling edge" ( ).
7. Place the three carts on the track.
    - Equip the cart on the left, which is closest to the starter system (hereinafter referred to as cart 1 with the velocity v_1), with a magnet with a plug facing the starter system and with a needle with a plug in the direction of motion.
    - Equip the cart in the middle (cart 2) and the cart on the right (cart 3) with a plasticine-filled tube facing cart 1 and with a needle with a plug in the direction of motion (see Fig. 5).

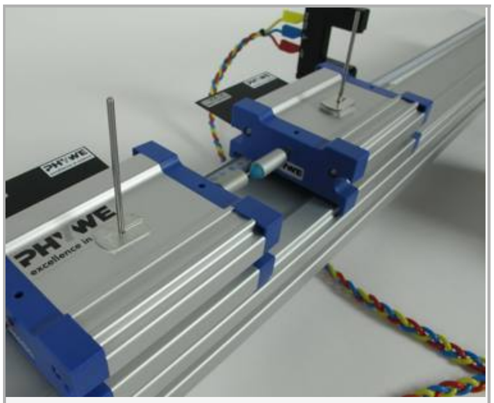

Figure 5: The collision is completed before the knocked-on cart enters the light beam of the light barrier.

    - Fasten a shutter plate (w = 100 mm) in all of the carts on the side where the light barriers are located.

Procedure

1. Prior to starting the measurement, determine the masses of the carts by way of the balance. The cart masses may vary slightly due to the different components that are attached to them. However, in this experiment, all three cart masses should be approximately identical at the beginning, which is why the 1 g weights should be used to perform small corrections, for example as shown in Figs. 2 and 5.
2. Set the timer to mode 1 "v(s)" (1 2 3 4). In this mode, the light barrier shading times are determined. Based on these values, and in combination with the known shutter plate length, the mean velocities with which the carts pass through the respective light barriers are calculated.
3. Prior to every collision experiment, press the "Reset" button in order to reset the displays.
4. Position cart 1 in the starter system. Cart 2 is located between light barriers 1 and 2, close to light barrier 2. Position cart 3 behind light barrier 2 just before light barrier 3 (compare Fig. 1). Ensure that the respective collision does not take place until the
rolling cart has completely passed through the previous light barrier, and that the collision is completed before the knocked-on cart enters the light beam of the next light barrier (see Fig. 5).
5. When the starter system is triggered, cart 1 is accelerated in the direction of cart 2. During this process, it receives an initial velocity v_1 and collides with cart 2. Both carts "stick together" and continue to move with the joint velocity v'. They then collide with cart 3 and pass jointly through the last light barrier with the velocity v''. Quantities with one superscript stroke have been measured after the first collision, and those with two superscript strokes after the second collision.
6. The light barriers only register the shading time of the shutter plate on the first cart. However, since the carts "stick together" and move with the same velocity, this shading time applies to all of them.
7. Use the three shading times t_i, together with the shutter plate length w = 100 mm, to determine the velocities $v_i = w/t_i$.
8. Record the measuring times during up to five repetitions and take the mean of these values. Then, repeat the measurement for different cart masses and mass ratios.

Observation and results

Observation

The carts collide successively with one another and continue to move in the same direction while being coupled to one another. The velocity decreases with every collision.

Evaluation

1. The cart masses and velocities of the measurements are used to calculate the momenta p_1 before the collisions and $p' = (m_1 + m_2) \cdot v'$ after the first and $p'' = (m_1 + m_2 + m_3) \cdot v''$ after the second collision. Since the carts are joined after the inelastic collision, they can be regarded as one cart with a greater mass.
2. In the case of central collisions of three masses, the law of conservation of momentum applies before and after every collision:
$$m_1 v_1 + m_2 v_2 + m_3 v_3 = (m_1 + m_2) v' + m_3 v_3 = (m_1 + m_2 + m_3) v'' \ . \qquad (1)$$
Based on the initial conditions $v_2 = v_3 = 0$, the law of conservation of momentum can be simplified as follows for this experiment:
$$m_1 v_1 = (m_1 + m_2) v' = (m_1 + m_2 + m_3) v'' \ . \qquad (2)$$
A comparison with the measured momenta (see the measurement example in Tables 1 and 2) confirms the law of conservation of momentum within the expected error limits.
The longer the distance that is covered is, the more the momentum of the carts decreases due to friction. In the measurement example, the total momentum between two light barriers decreases by approximately 2–4 %. In the case of high total masses, the deviations increase even more, which is due to the low cart velocities. This becomes particularly obvious during the measurement with three heavy carts of 800 g each, in which the total decrease in momentum is more than 15 %.
3. The kinetic energies E_1 before the collision and E' and E'' after the corresponding collisions are calculated. A comparison of the kinetic energies shows that the kinetic energy has clearly decreased after every collision (see Table 2). After two collisions, the kinetic energy has decreased rather significantly to less than 50 % of its original value (see $(E'' - E_1)/E_1$ in Table 2).
4. However, in a closed system, the energy is conserved and does not get lost. This discrepancy is due to the fact that, during the collisions, energy is used for deforming the plasticine putty. The total deformation energy $\Delta E_g = \Delta E' + \Delta E'' + \ldots$ is a combination of the deformation energies of the individual collisions and it causes the kinetic energy to decrease. As a result, the law of conservation of energy is:
$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 = \frac{1}{2}(m_1 + m_2) v'^2 + \Delta E' + \frac{1}{2} m_3 v_3^2 = \frac{1}{2}(m_1 + m_2 + m_3) v''^2 + \Delta$$
5. Under consideration of the law of conservation of momentum (2), the law of conservation of energy (3) yields the total deformation energy that can be expected
$$\Delta E_\mathrm{g} = -\frac{1}{2} m_1 v_1^2 \cdot \frac{m_2 + m_3}{m_1 + m_2 + m_3} \ . \qquad (4)$$
It depends solely on the individual masses and on the initial velocity. A comparison of the theoretical values with the measured values (Table 2) shows that they match very well.

Table 1: Measurement example with identical masses $m = m_1 = m_2 = m_3$.

m in kg	t_1 in s	t_2' in s	t_3'' in s
0.400	0.163	0.331	0.508
0.520	0.190	0.390	0.611
0.800	0.243	0.507	0.859

v_1 in m/s	*v*' in m/s	*v*'' in m/s	p_1 in kg m/s	*p*' in kg m/s	*p*'' in kg m/s
0.613	0.303	0.197	0.245	0.242	0.236
0.528	0.257	0.164	0.274	0.267	0.256
0.412	0.197	0.116	0.329	0.316	0.280

$(p'-p_1)/p_1$ in %	$(p''-p')/p'$ in %	$(p''-p_1)/p_1$ in %
−1.4	−2.4	−3.7
−2.7	−4.3	−6.9
−4.0	−11.5	−15.1

Tabelle 2: Messbeispiel für unterschiedliche Wagenmassen.

m_1 in kg	m_2 in kg	m_3 in kg	t_1 in s	t_2' in s	t_3'' in s
0.800	0.600	0.400	0.241	0.435	0.570
0.400	0.600	0.800	0.162	0.413	0.780
0.400	0.550	0.450	0.162	0.394	0.596
0.400	0.450	0.550	0.164	0.354	0.599
0.500	0.800	0.430	0.186	0.499	0.695

v_1 in m/s	v' in m/s	v'' in m/s	p_1 in kg m/s	p' in kg m/s	p'' in kg m/s
0.416	0.230	0.176	0.333	0.322	0.316
0.617	0.242	0.128	0.247	0.242	0.231
0.617	0.254	0.168	0.247	0.241	0.235
0.612	0.282	0.167	0.245	0.240	0.234
0.539	0.201	0.144	0.270	0.261	0.249

$(p'-p_1)/p_1$ in %	$(p''-p')/p'$ in %	$(p''-p_1)/p_1$ in %
−3.1	−1.9	−5.0
−1.9	−4.7	−6.5
−2.2	−2.7	−4.9
−1.9	−2.6	−4.4
−3.2	−4.5	−7.6

E_1 in kg m²/s²	E' in kg m²/s²	E'' in kg m²/s²	$\Delta E_g / E_1$ in %	$(E''-E_1)/E_1$ in %
0.0692	0.0371	0.0277	−56	−60
0.0762	0.0293	0.0148	−78	−81
0.0762	0.0307	0.0197	−71	−74
0.0748	0.0339	0.0195	−71	−74
0.0727	0.0262	0.0179	−71	−75

Note

1. In order to accelerate cart 1 with the starter system, the ram must be pushed in until it locks into place. Since the starter system offers three different levels, it must be ensured that the same locking position is used for all the experiments so that the same force is transferred.
2. The carts do not move completely without friction. There is still some residual friction and the total momentum decreases slightly. This also leads to a loss of energy so that the differences between the kinetic energies before and after the collisions do not correspond completely to the deformation energy ΔE of the plasticine putty.
3. The correct position of the shutter plates on the carts should be checked prior to every measurement, since they may be dislocated when the carts are abruptly stopped.
4. The plasticine should also be reshaped from time to time in order to buffer the impact of the cart to the highest possible extent.
5. The light barriers register only one interruption period. Fig. 6, for example, shows the situation immediately after the second collision. Light barrier 2 and light barrier 3 have been interrupted. While light barrier 3 registers the shading time of the leading cart, light barrier 2 is no longer active, since it has already registered the shading time of the middle cart before the second collision.

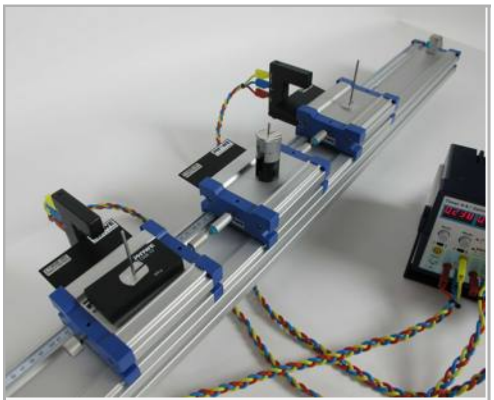

Figure 6: Light barrier interruption after the second collision